Math 220 – Homework 2

Due Thursday 9/13 at the beginning of class

Total points: 165 (Problems marked by $*$ will count toward writing portion.)

PART A

Problems from the textbook:

- Section 1.1

problem	2(e)*	3(a,c)*	10(a,b)	12(a,b)	14(b,c)
points	5	10	10	10	10

PART B

1. [8 points] In each of the following statements identify the hypothesis (assumption) and conclusion.
   (a) If a is irrational, then $2a$ is irrational.
   (b) a^3 is an even integer whenever a is an even integer.

2. [12 points] Without changing their meanings, convert each of the following sentences into a sentence having the form "If P, then Q."
   (a) A function is integrable provided the function is continuous.
   (b) A function is rational if it is a polynomial.
   (c) "You fail only if you stop writing." (Ray Bradbury)
   (d) "Whenever people agree with me I feel I must be wrong." (Oscar Wilde)

3. [4 points] Without changing its meaning, convert the sentence *"If $xy = 0$, then $x = 0$ or $y = 0$, and conversely."* into a sentence having the form "P if and only if Q."

   If a function has a constant derivative, then it is linear, and conversely.

4. [4 points] Write the following statement using "if, then":

   "A sufficient condition for a triangle to be isosceles is that it has two equal angles."

5. * [8 points] Prove that the statement $\neg((\neg Q \wedge (P \Rightarrow Q)) \Rightarrow (\neg P))$ is a tautology, a contradiction, or neither. You must state which of the three it is as well as give the proof.

6. [25 points] Negate the following statements:
   (a) Every real number is less than 100.
   (b) There is a politician who is honest or trustworthy.
   (c) If f is a linear function, then f is continuous at 0.
   (d) If a differentiable function f has a local minimum at the point x_0, then $f'(x_0) = 0$.

7. Given a quantified statement
$$\forall x \in \mathbb{Z}^+, (\exists y \in \mathbb{Z}^+ \ni xy \in \mathbb{E}). \tag{1}$$
   (a) [3 points] Express the given statement (1) in words.
   (b) [6 points] Express the **negation** of the given statement (1) in symbols. **(Do NOT use the symbol "$\notin$".)**
   (c) [3 points] Express the **negation** of the given statement (1) in words.

8. Given a quantified statement
$$\forall x \in \mathbb{R},\ \exists n \in \mathbb{Z}\ \ni\ (n \leq x < n + 1). \tag{2}$$
   (a) [3 points] Express the statement (2) in words.

(b) [8 points] Express the **negation** of the statement (2) in symbols. **(Do NOT use the symbol "$\notin$" and interval notation.)**

9. [36 points] Express the following statements in the form *"For all . . . , if . . . then. . . "* using symbols to represent variables. Then write their negations in words, again using symbols to represent variables. (**Attention you should use symbols to represent introduced variables only. The statements and their negations must be written in words and not in symbols.**)

   (a) An integer is odd or even.

   (b) All angles of a square are equal.

   (c) The number -1 is the largest negative integer.

   (d) When the product of two integers is odd, then the both integers are odd.

   (e) Every multiple of 6 is even and is not a multiple of 4.

   (f) The square of an even integer is divisible by 4.