Math 220 – Homework 2

Due Friday 02/06 at the beginning of class

Total points: 197

PART A

Problems from the textbook:

- Section 1.2

problem	1(b)*	2(a,b)*	4*
points	10	20	10

PART B

1. [12 points] Consider the following statement:

   "If $\sqrt{3} < \sqrt{7}$, then $3 < 7$."

   Write in a useful form

   (a) the converse;

   (b) the contrapositive;

   (c) the converse of contrapositive;

   (d) the contrapositive of converse.

2. [16 points] Write the converse and contrapositive of each statement.

   (a) For every real x, if $x > 1$ or $x < -1$, then $x^4 > 1$.

   (b) For every integer n, n^2 is a multiple of 3 is sufficient for n to be a multiple of 3.

   (c) The sequence $\{a_n\}$ converges if $\{a_n\}$ is bounded and monotonic.

3. [12 points] Express the following statements in symbols. (**Do not use** "$\Rightarrow$".)

   (a) Every even integer can be expressed as the sum of two odd integers.

   (b) The square of any real number is positive.

   (c) Every prime number is greater than 1.

4. [25 points] Negate the following statements:

   (a) Every prime number is greater than 1.

   (b) There are sets that contain infinitely many elements.

   (c) There is a cold medication that is safe and effective.

   (d) The number p is prime or the number q is not prime.

   (e) If a differentiable function f has a local minimum at the point x_0, then $f'(x_0) = 0$.

5. Given a quantified statement

$$\forall n \in \mathbb{O},\ \exists x \in \mathbb{Z}\ \ni\ n = 4x + 1\ \vee\ n = 4x + 3. \tag{1}$$

   (a) [3 points] Express the statement (2) in words.

   (b) [6 points] Express the **negation** of the statement (2) in symbols. **(Do NOT use the symbol $\notin$.)**

6. Given a quantified statement

$$\forall a \in \mathbb{R},\ \exists n \in \mathbb{Z}\ \ni\ a \in (n-1, n]. \tag{2}$$

(a) [3 points] Express the statement (2) in words.

(b) [8 points] Express the **negation** of the statement (2) in symbols. **(Do NOT use the symbol "$\notin$" and interval notation.)**

7. Consider the following statement:

   "If x is a real positive number, then there is a real positive number ε such that $x < \varepsilon$ but $\frac{1}{\varepsilon} < x$."

   (a) [3 points] Express the given statement in symbols. (**Do not use** "$\Rightarrow$")

   (b) [7 points] Express the **negation** of the given statement in symbols.

   (c) [3 points] Express the **negation** of the given statement in words.

8. [36 points] Express the following statements in the form *"For all ..., if ... then..."* using symbols to represent variables. Then write their negations in <u>words</u>, again using symbols to represent variables.

   (a) An integer is odd or even.

   (b) An angles of a square are equal.

   (c) The number -1 is the largest negative integer.

   (d) When the product of two integers is odd, then the both integers are odd.

   (e) Every multiple of 6 is even and is not a multiple of 4.

   (f) The square of an even integer is divisible by 4.

9. * [10 points] Let $x \in \mathbf{R}$. Prove that if $0 < x < 1$, then $x^2 - 2x + 2 \neq 0$.

10. * [10 points] Let $z \in \mathbf{R}^+$. Prove that if $z^4 - 2z^2 + 2 \leq 0$, then $z^{2018} \geq 2018$.