Math 220 – Homework 3 (HNR)

Due Thursday 9/20 at the beginning of class

Total points: 136 (Problems marked by $*$ will count toward writing portion.)

PART A

Problems from the textbook:

problem	3.10*	2.80(a)*	2.82 *	2.84*
points	10	10	15	10

It is advised to read Section 2.11 in the textbook.

PART B

1. [18 points] Let n represents some fixed integer. In each of the following statements identify the hypothesis (assumption) and conclusion.
   (a) The number n divides 5 only if n divides 10,
   (b) The condition $n^2 \in 3\mathbb{Z}$ is necessary for n to be a multiple of 3.
   (c) The condition $n \in \mathbb{E}$ is sufficient for n to be a multiple of 4.
2. * [10 points] Let $x \in \mathbf{R}$. Prove that if $|x| < 1$, then $x^4 - 2x^2 + 2 \neq 0$.
3. * [10 points] Let $x \in \mathbf{R}^+$. Prove that if $x^2 - 2x + 2 \leq 0$, then $x^{2019} \geq 2019$.
4. * [10 points] Prove that if n is an even integer, then $n^{2019} + 19(n-1)^2 - 2019$ is even. (Give a formal proof).
5. Consider the following statement:

   ``For all integers x and y such that $x \neq 0$, if $x|y$, then $x^{17}|y^{17}$.''

   (a) [10 points] Prove the above statement.
   (b) [3 points] Formulate the converse statement.
6. [12 points] For the statement
   S: ``For every integer n, if n is divisible by 3 and n is divisible by 5, then n is divisible by 15.'' write in a useful form
   (a) the converse of S;
   (b) the contrapositive of S.
   (c) the converse of the contrapositive of S;
   (d) the contrapositive of the converse of S.
7. Consider the following definition:

   A real-valued function $f(x)$ is said to be **decreasing** *on the closed interval $[a, b]$, if for all $x_1, x_2 \in [a, b]$, if $x_1 < x_2$, then $f(x_1) > f(x_2)$.*

   (a) [6 points] Write the negation of this definition completing the following: "A real-valued function $f(x)$ is said to be **not decreasing** on the closed interval $[a, b]$, if ..."
   (b) [6 points] Give an example of a decreasing function on $[-1, 1]$ and based on the above definition explain why your example is correct.
   (c) [6 points] Give an example of a function that is not decreasing on $[-1, 1]$ and based on the negation of the above definition explain why your example is correct.