

Math 220 – Homework 3

point split: Total 120 Points

Due Wednesday 09/22 at the beginning of class

PART A

Problems from the textbook:

- Section 1.4 # 5, 7, 20

PART B

1. Let $x \in \mathbf{R}$. Prove that if $x^2 - 2x + 2 \leq 0$, then $x^{2016} \geq 2016$.
2. Prove that if n is an even integer, then $n^{2016} + (n - 1)^2 - 17$ is even.
3. Prove that if x and y are odd integers, then $xz - yz$ is even for every integer z .
4. Prove the following statement:

‘‘Let $n \in \mathbf{Z}$. Then n is odd if and only if $7n + 17$ is even.’’

5. Prove that if $n \in \mathbf{Z}$, then $n^3 + n$ is even.
6. Prove that $5x - 3y$ is even, then x and y are of the same parity.
7. Consider the following statement:

‘‘For all integers x and y such that $x \neq 0$, if $x|y$, then $x^5|y^5$.’’

- (a) Prove the above statement.
 - (b) Formulate the converse statement.
 - (c) If the converse statement is true, prove it. Otherwise, explain why it is false.
8. (a) Let $n \in \mathbf{Z}$. Prove that if $2|(n^2 - 5)$, then $4|(n^2 - 5)$.
(b) Give an example of an integer n such that $2|(n^2 - 5)$, but $8 \nmid (n^2 - 5)$.