Math 220 – Homework 3

Due Friday 2/13 at the beginning of class

Total points: 171

PART A

Problems from the textbook:

- Section 2.1

problem	1*	4*	5(a,b,e,f,g)*
points	10	20	30

PART B

1. * [10 points] Prove that if x and y are odd integers, then $xz - yz$ is even for every integer z.

2. * [20 points] Let a and b be integers. Prove the following statements.
   (a) If $a|b$, then $a|(b^4 - b^2 + 5b)$.
   (b) If $a^2|a$, then $a \in \{-1, 0, 1\}$.

3. * [20 points] Prove or disprove the following statements:
   (a) For all real numbers a and b, $\sqrt{a^2+b^2} = \sqrt{a^3+b^3}$.
   (b) Let $n \in \mathbf{Z}$. If $n^2 + 3n$ is even, then n is odd.
   (c) If $n \in \{0, 1, 2, 3, 4\}$, then $2^n + 3^n + n(n-1)(n-2)$ is prime.
   (d) For every integer n, if $n \in \mathbb{E}$ and $n \in 6\mathbb{Z}$, then n is divisible by 12.

4. Consider the following statement:

   ``For all integers a and b such that $a \neq 0$, if $a|b$, then $a^7|b^7$.''

   (a) * [10 points] Prove the above statement.
   (b) [3 points] Formulate the converse statement.

5. [12 points] For the statement
   S: ``For every integer n, if n is divisible by 3 and n is divisible by 5, then n is divisible by 15.'' write in a useful form
   (a) the converse of S;
   (b) the contrapositive of S.
   (c) the converse of the contrapositive of S;
   (d) the contrapositive of the converse of S.

6. Consider the following definition:

   A real-valued function $f(x)$ is said to be **monotonically decreasing** *on the closed interval I, if for all $x_1, x_2 \in I$, if $x_1 < x_2$, then $f(x_1) > f(x_2)$.*

   (a) [6 points] Write the negation of this definition completing the following: "A real-valued function $f(x)$ is said to be **not monotonically decreasing** on the closed interval I, if ..."
   (b) [6 points] Give an example of a monotonically decreasing function on the interval $[-1, 1]$ and based on the above definition explain why your example is correct.

(c) [6 points] Give an example of a function that is not monotonically decreasing on the interval $[-1, 1]$ and based on the negation of the above definition explain why your example is correct.

7. [**18 points**] Let n represents some fixed integer. In each of the following statements identify the hypothesis (assumption) and conclusion.

(a) The number n divides 5 only if n divides 10,

(b) The condition $n^2 \in 3\mathbb{Z}$ is necessary for n to be a multiple of 3.

(c) The condition $n \in \mathbb{E}$ is sufficient for n to be a multiple of 4.