

Math 220 (902&906) – Homework 4

Due Wednesday 09/29 at the beginning of class

PART A

Problems from the textbook:

- Section 1.3 # 1(b,c), 2(a,b,c), 4, 8
- Section 1.4 # 17, 21¹

PART B

1. Disprove the following statements:

- (a) For all positive integers x, y, z , $x^{y^z} = (x^y)^z$.
- (b) If $n \in \{0, 1, 2, 3, 4\}$, then $2^n + 3^n + n(n-1)(n-2)$ is prime.
- (c) There exist odd integers a and b such that $4|(7a^2 - b^2)$. (Hint: Use proof by contradiction.)

2. Prove the statement ‘‘If n is an odd integer, then $27n + 5$ is even.’’ by

- (a) a direct proof;
- (b) a proof by contrapositive;
- (c) a proof by contradiction.

3. Let $x, y \in \mathbf{R}$. Proof that if $xy \neq 0$, then $x \neq 0$ by using more than one method of proof.

4. Use proof by contradiction to prove that if a and b are odd integers, then $4 \nmid (a^2 + b^2)$.

5. Consider the following statement:

‘‘If $\sqrt{3} < \sqrt{7}$, then $3 < 7$.’’

Write in a useful form

- (a) the converse;
- (b) the contrapositive;
- (c) the converse of contrapositive;
- (d) the contrapositive of converse.

¹Hint: use problem 20 from the section 1.4