

Math 220 – Homework 4

Due Thursday 9/27 at the beginning of class

Total points: 170 (Problems marked by * will count toward writing portion.)

PART A

Problems from the textbook:

• Section 2.1	problem	1*	4*	5(a,b,e,f,g)*
	points	10	20	30

PART B

- * [40 points] Prove or disprove the following statements:
 - For all positive integers x, y, z , $x^{y^z} = (x^y)^z$.
 - If $n \in \{0, 1, 2, 3, 4\}$, then $2^n + 3^n + n(n-1)(n-2)$ is prime.
 - For every integer n , if n is divisible by 2 and n is divisible by 6, then n is divisible by 12.
 - There exists an odd integer n such that $n^2 + 2n + 3$ is odd.
- * [10 points] Let $a, b \in \mathbb{Z}$. Prove that if $a|b$ and $b|a$, then $a = b$ or $a = -b$.
- * [10 points] Prove that for every integer x , the integers $3x + 1$ and $5x + 2$ are of opposite parity.
- * [10 points] Let $x \in \mathbb{Z}$. Prove that if $7x + 5$ is odd, then x is even.
- * [10 points] Let $x, y \in \mathbb{Z}$. Prove that if xy is odd, then x and y are odd.
- * [10 points] Let $x, y \in \mathbb{Z}$. Prove that if $3x + 7y$ is even, then x and y are of the same parity.
- * [10 points] Let $a, b, c \in \mathbb{Z}$. Prove that if $a \nmid bc$, then $a \nmid b$ and $a \nmid c$.
- * [10 points] Prove that if n is an odd integer, then $8|(n^2 + (n+6)^2 + 6)$.