Math 220 – Homework 4

Due Friday 2/20 at the beginning of class

Total points: 157

PART A

Problems from the textbook:

- Section 2.2

problem	2*	3*	5*	8*
points	10	10	10	10

PART B

1. [5 points] Write the contrapositive and converse of the following statement
   "*For all integers* a, b, *and* c, *if* $a|(b+c)$, *then* $a|b$ *or* $a|c$."
2. * [10 points] Prove that $7a + 5b \in 2\mathbb{Z}$, then a and b are of the same parity.
3. Consider the statement:

   "If the product of two integers is even, then at least one of these integers is even."

   (a) [2 points] Rewrite the statement in symbols.
   (b) * [8 points] Give a formal proof.
4. * Let n be an integer.
   (a) [10 points] Prove that n is even if and only if n^3 is even.
   (b) [2 points] Prove that n is odd if and only if n^3 is odd.
   (c) [10 points] Prove that $\sqrt[3]{2}$ is irrational.
5. * [10 points] Let $x, y \in \mathbf{R}$. Proof that if $xy \neq 0$, then $x \neq 0$.
6. * [10 points] Prove that if $x, y \in \mathbb{O}$, then $x^2 + y^2 \notin 4\mathbb{Z}$.
7. (a) * [10 points] Let $n \in \mathbf{Z}$. Prove that if $2|(n^2 - 5)$, then $4|(n^2 - 5)$.
   (b) [5 points] Give an example of an integer n such that $2|(n^2 - 5)$, but $8 \nmid (n^2 - 5)$
8. * Prove the statement `‘‘If` n `is an odd integer, then` $7n - 5$ `is even.’’` by
   (a) [5 points] a direct proof (give a formal proof);
   (b) [5 points] a proof by contrapositive (give a formal proof);
   (c) [5 points] a proof by contradiction (give a formal proof).

   To submit the next question it is recommended to print page 2 of the current assignment and attach it to your homework.
9. [20 points] You have been introduced to three proof techniques: direct proof, proof by contrapositive and proof by contradiction. The attached table (see next page) gives several ways that we might start the proof. Only some of these can lead to a proof, however. Fill in the table. For each step in the column "Technique" write one of the following

   - direct proof
   - proof by contrapositive
   - proof by contradiction
   - A mistake has been made.

   If a mistake has been made, leave blank the corresponding "Goal" space, otherwise write what is the goal.

How to prove (and not to prove) that $\forall x \in D, P(x) \Rightarrow Q(x)$.

	First step of "Proof"	**Technique**	**Goal**
1	Assume that there exists $x \in D$ such that $P(x)$ is true.		
2	Assume that there exists $x \in D$ such that $P(x)$ is false.		
3	Assume that there exists $x \in D$ such that $Q(x)$ is true.		
4	Assume that there exists $x \in D$ such that $Q(x)$ is false.		
5	Assume that there exists $x \in D$ such that $P(x)$ and $Q(x)$ are true.		
6	Assume that there exists $x \in D$ such that $P(x)$ is true and $Q(x)$ is false.		
7	Assume that there exists $x \in D$ such that $P(x)$ is false and $Q(x)$ is true.		
8	Assume that there exists $x \in D$ such that $P(x)$ and $Q(x)$ are false.		
9	Assume that there exists $x \in D$ such that $P(x) \Rightarrow Q(x)$ is true.		
10	Assume that there exists $x \in D$ such that $P(x) \Rightarrow Q(x)$ is false.		