

Math 220 – Homework 5

Due Thursday 10/04 at the beginning of class

Total points: 132 (Problems marked by * will count toward writing portion.)

PART A*

Problems from the textbook:

• Section 2.2	problem	3*	5*
	points	10	10

PART B

1. * [10 points] Let $a, b \in \mathbf{R}$. Prove that if $ab \neq 0$, then $a \neq 0$ and $b \neq 0$.
2. * [10 points] Prove that if a , and b are odd integers, then $4 \nmid (a^2 + b^2)$.
3. * Let n be an integer.
 - (a) [10 points] Prove that n is even if and only if n^3 is even.
 - (b) [2 points] Prove that n is odd if and only if n^3 is odd.
 - (c) [10 points] Prove that $\sqrt[3]{2}$ is irrational.
4. (a) * [10 points] Let $n \in \mathbf{Z}$. Prove that if $2 \mid (n^2 - 5)$, then $4 \mid (n^2 - 5)$.
 (b) [5 points] Give an example of an integer n such that $2 \mid (n^2 - 5)$, but $8 \nmid (n^2 - 5)$
5. Consider the statement:

“If the product of two integers is even, then at least one of these integers is even.”

 - (a) [2 points] Rewrite the statement in symbols.
 - (b) * [8 points] Give a formal proof.
6. * [10 points] Prove that there are no integers m and n such that $m^2 = 4n + 3$.
7. * [10 points] Let $a, b \in \mathbf{Z}$ such that $a \geq 2$. Prove that $a \nmid b$ or $a \nmid (b + 1)$.
8. * [10 points] Prove that there is no largest negative rational number.
9. * Prove the statement “If n is an odd integer, then $3 - 5n$ is even.” by
 - (a) [5 points] a direct proof (give a formal proof);
 - (b) [5 points] a proof by contrapositive (give a formal proof);
 - (c) [5 points] a proof by contradiction (give a formal proof).