Math 220 – Homework 5

Due Wednesday 02/24 at the beginning of class

1. Prove that the equation $x^5 + 2x - 5 = 0$ has a *unique* real number solution between $x = 1$ and $x = 2$.

2. Prove that the equation $\sin^{2016}(x) - 4x + \pi = 0$ has a real number solution between $x = 0$ and $x = 4$. (You may assume that $\sin^{2016}(x)$ is continuous on $[0, 4]$.)

3. Let $a, b, c \in \mathbf{Z}$. Determine the truth or falsehood of the following statements. If the statement is true, provide a reasoning; otherwise, disprove the statement.

   (a) $0|b$ only if $b = 0$.

   (b) If $a|c$ and $b|c$, then $ab|c$.

   (c) If $a|b$ and $b|a$ then $a = b$.

4. Prove by induction that for every positive integer n the following statements hold:

   (a) $2 + 6 + 10 + \ldots + (4n - 2) = 2n^2$.

   (b) $n^3 + 2n$ is divisible by 3. (Hint: $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$)

   (c) $\dfrac{1}{2 \cdot 3} + \dfrac{1}{3 \cdot 4} + \ldots + \dfrac{1}{(n + 1)(n + 2)} = \dfrac{n}{2(n + 2)}$.

   (d) $7|(2^{3n} - 1)$.

   (e) 3 is a factor of $7^n - 4^n$.