# Math 220 (HNR) – Homework 6

## PART A*

Problems from the textbook:

- # 6.5, 6.10, 6.21, 6.23, 6.50

## PART B*

1. Prove that the equation $x^5 + 2x - 5 = 0$ has a *unique* real number solution between $x = 1$ and $x = 2$.
2. Prove that the equation $\sin^{2018}(x) - 4x + \pi = 0$ has a real number solution between $x = 0$ and $x = 4$. (You may assume that $\sin^{2018}(x)$ is continuous on $[0, 4]$. )
3. Prove by induction that if $n$ is a positive integer, then $9^n - 4^n \in 5\mathbb{Z}$.
4. Prove by induction that for every positive integer $n$ the following statements hold:
   (a) $n^3 + 8n + 9$ is divisible by 3.
   (b) $\dfrac{1}{2 \cdot 3} + \dfrac{1}{3 \cdot 4} + \ldots + \dfrac{1}{(n+1)(n+2)} = \dfrac{n}{2(n+2)}$.
   (c) $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$