

Math 220 – Homework 6

Due Tuesday 03/20 at the beginning of class

Total points: 237

PART A

Problems from the textbook:

• Section 4.1	problem	4*	5*	6*
	points	10	10	10

• Section 4.2	problem	1(f)*	11	15	16(c)	18	23(b)
	points	10	5	8	4	15	10

PART B

- * [15 points] Let $A, B,$ and C be nonempty subsets of a universal set U . Disprove the following statements:
 - If $A \cap B = A \cap C$, then $B = C$.
 - If $A - B = C - B$, then implies $A = C$.
 - If A is not a subset of B and B is not a subset of A , then $A \cap B = \emptyset$.
- [6 points] Describe the following sets by listing their elements.
 - The set of all remainders when a positive integer is divided by 8.
 - The set of all integers of absolute value less than 1.
- [6 points] Describe the following sets by listing enough elements to indicate a pattern for all elements of the set.
 - The set of all remainders when a natural number is divided by 2018.
 - The set of all numbers x for which $\sin x = 1$.
- [6 points] Describe the following sets using a set-builder notation. Namely, write them in the form $\{x \in D \mid \dots\}$ for the appropriate set D .
 - The set of all rational numbers between 0 and 1 inclusive.
 - The set of all numbers x for which $\tan x = 0$.
- [12 points] Let $U = \mathbb{R}$ be the universal set. Consider $A = \{x \in \mathbb{R} \mid |2x + 3| \geq 19\}$ and $B = \{x \in \mathbb{R} \mid |x| \leq 3\}$.
 - Express the sets A and B using interval notation (as an interval or a union of intervals).
 - Determine $\overline{A \cap B}$ as an interval or a union of intervals.
- [15 points] Let $U = \{x, y, 1, 2, 3\}$ be the universal set and let $A = \{x, y, 1, 2\}$, $B = \{2, 3\}$, $C = \{1, 3, x, y\}$. Determine the following (show all intermediate steps):
 - $\overline{A \cup (B \cap C)}$
 - $\overline{B \cup C} \cap U$
 - $\overline{(A \cup B) - (B \cap C)}$
- [10pts] Given $A = \{x \in \mathbb{Z} \mid |x| > 10\}$. Compute the compliment of A , if
 - $U = \mathbb{Z}$.
 - $U = \mathbb{R}$.
- [10pts] Given $A = \{x \in \mathbb{R} \mid |x| > 10\}$ and $B = \{x \in \mathbb{R} \mid 0 < |x| \leq 12\}$. Compute $A - B$ and $B - A$.

PART C

Print it out and attach to the Homework

1. [12 points] Determine the truth or falsehood of the following statements. (CLEARLY circle TRUE or FALSE for each statement.)
- (a) TRUE/ FALSE The contrapositive of the statement
 “If all elements of A are elements of B , then A is a subset of B ”
 is the statement
 “If A is a subset of B , then all elements of A are elements of B ”.
- (b) TRUE/ FALSE $\{a, b\} = \{b, a, b\}$
- (c) TRUE/ FALSE If $A = \{m \in \mathbb{Z} \mid 2 < m \leq 5\}$ then $|A| = 4$.
- (d) TRUE/ FALSE The empty set is a subset of every set except itself.
- (e) TRUE/ FALSE $7 \notin \{-1, 7\}, \{-7, 2017, 0\}, \{1, 2\}$.
- (f) TRUE/ FALSE If $A = \{a, \{a, b, c\}\}$ and $B = \{\{c, d\}, \{a, b, c, d\}\}$ then $|A| = |B|$.
2. [19 points] Given $A = \{0, 1, 2, \dots, 8\}$, $B = \{1, 3, 5, 7\}$, $C = \{3, 5, 1, 7, 3, 1\}$,
 $D = \{5, 3, 1\}$, and $E = \emptyset$. Determine the truth or falsehood of the following statements. (CLEARLY circle T (TRUE) or F (FALSE) for each statement.)
- (a) T/F $B = C$ (b) T/F $B \subseteq C$ (c) T/F $B \subset C$ (d) T/F $C \subseteq B$ (e) T/F $D \subset B$
- (f) T/F $D \subseteq B$ (g) T/F $B \subset D$ (h) T/F $8 \in A$ (i) T/F $\{4, 6\} \subset A$ (j) T/F $1, 5 \subset A$
- (k) T/F $9 \notin C$ (l) T/F $D \subseteq D$ (m) T/F $\emptyset = 0$ (n) T/F $0 \in E$ (o) T/F $A \in A$
- (p) T/F $|A| = 8$ (q) T/F $|C| = 7$ (r) T/F $|E| = 0$ (s) T/F $|B| = 5$
3. [14pts] Let A, B , and C be nonempty sets. Determine the truth or falsehood of the following statements. (CLEARLY circle TRUE or FALSE for each statement.)
- (a) TRUE/ FALSE $A - A = \emptyset$.
- (b) TRUE/ FALSE $A \subset A$.
- (c) TRUE/ FALSE $A \cup (B \cap C) = (A \cap C) \cup (B \cap C)$.
- (d) TRUE/ FALSE $A \cup A = A \cap A$ for all sets A .
- (e) TRUE/ FALSE If $|A| = |B|$ then $A \times B = B \times A$.
- (f) TRUE/ FALSE $A \times B = B \times A$ for all nonempty sets A and B .
- (g) TRUE/ FALSE If $\{1\} \in P(A)$, then $1 \in A$ and $\{1\} \notin A$.
4. [10pts] Let $A = \{x \in \mathbb{N} \mid 1 \leq x < 5\}$ and $P(A)$ be a power set of A . Determine the truth or falsehood of the following statements. (CLEARLY circle TRUE or FALSE for each statement.)
- (a) TRUE/ FALSE $A \subset P(A)$.
- (b) TRUE/ FALSE $\{2\} \in P(A)$.
- (c) TRUE/ FALSE $[3, 4] \subseteq A$.
- (d) TRUE/ FALSE $|P(A)| = 32$
- (e) TRUE/ FALSE $\emptyset \subseteq P(A)$ and $\emptyset \in P(A)$.

5. Assume that A and B are sets and P and Q are propositions. Characterize the following expressions as either

- (i) a proposition
- (ii) not a proposition, but an expression that makes sense mathematically.
- (iii) an expression that makes no sense mathematically.

(CLEARLY circle i, ii, or iii for each statement.)

- (a) $B \subset A$
- (b) $A \Rightarrow B$
- (c) $P \Rightarrow (A = B)$
- (d) $P \subseteq Q$
- (e) $P = Q$
- (f) $B \cap (A - B)$
- (g) $P \cup Q$
- (h) $A + B$
- (i) $(\exists x \in P)[x \in Q]$
- (j) $A \wedge B$