Math 220 HNR – Homework 7

Due Thursday 10/25 at the beginning of class

Total points: 295

PART A

Problems from the textbook:

problem	1.4(a,b,c,d)	1.8	1.12	1.22	1.28	1.46	1.48
points	12	25	5	18	10	30	10

Read section 1.5.

PART B

1. * [10 points] Let A, B, and C be subsets of some universal set U. Prove that $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$.
2. * [10 points] Consider the sets $A = \{n \in \mathbb{Z}|\ \exists k \in \mathbb{Z} \ni n = 12k + 11\}$ and $B = \{n \in \mathbb{Z}|\ \exists k \in \mathbb{Z} \ni n = 4k + 3\}$.
   (a) Is $A \subseteq B$? Prove or disprove.
   (b) Is $B \subseteq A$? Prove or disprove.
3. * [10 points] Consider the sets $A = \{n \in \mathbb{Z}|\ \exists k \in \mathbb{Z} \ni n = 3k\}$ and $B = \{n \in \mathbb{Z}|\ \exists i, j \in \mathbb{Z} \ni n = 15i + 12j\}$. Prove that $A = B$.
4. * [10 points] Consider the sets $A = \{n \in \mathbb{Z}|\ \exists k \in \mathbb{Z} \ni n = 4k + 1\}$ and $B = \{n \in \mathbb{Z}|\ \exists k \in \mathbb{Z} \ni n = 4k - 7\}$. Prove that $A = B$.
5. * [15 points] Let A, B, and C be nonempty subsets of a universal set U. Disprove the following statements:
   (a) If $A \cap B = A \cap C$, then $B = C$.
   (b) If $A - B = C - B$, then implies $A = C$.
   (c) If A is not a subset of B and B is not a subset of A, then $A \cap B = \emptyset$.
6. [6 points] Describe the following sets by listing their elements.
   (a) The set of all reminders when a positive integer is divided by 8.
   (b) The set of all integers of absolute value less than 1.
7. [6 points] Describe the following sets by listing enough elements to indicate a pattern for all elements of the set.
   (a) The set of all reminders when a natural number is divided by 2018.
   (b) The set of all numbers x for which $\sin x = 1$.
8. [6 points] Describe the following sets using a set-builder notation. Namely, write them in the form $\{x \in D|\ \ldots\}$ for the appropriate set D.
   (a) The set of all rational numbers between 0 and 1 inclusive.
   (b) The set of all numbers x for which $\tan x = 0$.
9. [12 points] Let $U = \mathbb{R}$ be the universal set. Consider $A = \{x \in \mathbb{R}|\,|2x + 3| \geq 19\}$ and $B = \{x \in \mathbb{R}|\,|x| \leq 3\}$.
   (a) Express the sets A and B using interval notation (as an interval or a union of intervals).
   (b) Determine $\overline{A} \cap \overline{B}$ as an interval or a union of intervals.
10. [10 points] Given $A = \{x \in \mathbb{Z}|\ \ |x| > 10\}$. Compute the compliment of A, if (a) $U = \mathbb{Z}$ (b) $U = \mathbb{R}$.
11. [10 points] Given $A = \{x \in \mathbb{R}|\ \ |x| > 10\}$ and $B = \{x \in \mathbb{R}|\ 0 < |x| \leq 12\}$. Compute $A - B$ and $B - A$.
12. [5 points]Let $A = \{a, b\}$, $B = \{1, 2\}$, and $C = \{c, d, e\}$. Explain why $A \times (B \times C) \neq (A \times B) \times C$.

PART C **Print it out and attach to the Homework**

NAME (print)______________________________

1. [12 points] Determine the truth or falsehood of the following statements. (CLEARLY circle TRUE or FALSE for each statement.)

   (a) TRUE/ FALSE The contrapositive of the statement

   "If all elements of A are elements of B, then A is a subset of B"

   is the statement

   "If A is a subset of B, then all elements of A are elements of B".

   (b) TRUE/ FALSE $\{a, b\} = \{b, a, b\}$

   (c) TRUE/ FALSE If $A = \{m \in \mathbb{Z} |\ 2 < m \leq 5\}$ then $|A| = 4$.

   (d) TRUE/ FALSE The empty set is a subset of every set except itself.

   (e) TRUE/ FALSE $7 \notin \{\{-1, 7\}, \{-7, 2017, 0\}, \{1, 2\}\}$.

   (f) TRUE/ FALSE If $A = \{a, \{a, b, c\}\}$ and $B = \{\{c, d\}, \{a, b, c, d\}\}$ then $|A| = |B|$. .

2. [19 points] Given $A = \{0, 1, 2, \ldots, 8\}$, $B = \{1, 3, 5, 7\}$, C [20pts] $= \{3, 5, 1, 7, 3, 1\}$, $D = \{5, 3, 1\}$, and $E = \emptyset$. Determine the truth or falsehood of the following statements.(CLEARLY circle T (TRUE) or F (FALSE) for each statement.)

   (a) T/F $B = C$ **(b)** T/F $B \subseteq C$ **(c)** T/F $B \subset C$ **(d)** T/F $C \subseteq B$ **(e)** T/F $D \subset B$

   (f) T/F $D \subseteq B$ **(g)** T/F $B \subset D$ **(h)** T/F $8 \in A$ **(i)** T/F $\{4, 6\} \subset A$ **(j)** T/F $1, 5 \subset A$

   (k) T/F $9 \notin C$ **(l)** T/F $D \subseteq D$ **(m)** T/F $\emptyset = 0$ **(n)** T/F $0 \in E$ **(o)** T/F $A \in A$

   (p) T/F $|A| = 8$ **(q)** T/F $|C| = 7$ **(r)** T/F $|E| = 0$ **(q)** T/F $|B| = 5$

3. [14 points] Let A, B, and C be nonempty sets. *Determine the truth or falsehood of the following statements.* (CLEARLY circle TRUE or FALSE for each statement.)

   (a) TRUE/ FALSE $A - A = \emptyset$.

   (b) TRUE/ FALSE $A \subset A$.

   (c) TRUE/ FALSE $A \cup (B \cap C) = (A \cap C) \cup (B \cap C)$.

   (d) TRUE/ FALSE $A \cup A = A \cap A$ for all sets A.

   (e) TRUE/ FALSE If $|A| = |B|$ then $A \times B = B \times A$.

   (f) TRUE/ FALSE $A \times B = B \times A$ for all nonempty sets A and B.

   (g) TRUE/ FALSE If $\{1\} \in P(A)$, then $1 \in A$ and $\{1\} \notin A$.

4. [10 points] Let $A = \{x \in \mathbb{N} |\ 1 \leq x < 5\}$ and $P(A)$ be a power set of A. Determine the truth or falsehood of the following statements. (CLEARLY circle TRUE or FALSE for each statement.)

   (a) TRUE/ FALSE $A \subset P(A)$.

   (b) TRUE/ FALSE $\{2\} \in P(A)$.

   (c) TRUE/ FALSE $[3, 4] \subseteq A$.

   (d) TRUE/ FALSE $|P(A)| = 32$

   (e) TRUE/ FALSE $\emptyset \subseteq P(A)$ and $\emptyset \in P(A)$.

5. [20 points] Assume that A and B are sets and P and Q are propositions. Characterize the following expressions as either

(i) a proposition/statement

(ii) not a proposition/statement, but an expression that makes sense mathematically.

(iii) an expression that makes no sense mathematically.

(CLEARLY circle i, ii, or iii for each statement.)

(a) i / ii /iii $B \subset A$

(b) i / ii /iii $A \Rightarrow B$

(c) i / ii /iii $P \Rightarrow (A = B)$

(d) i / ii /iii $P \subseteq Q$

(e) i / ii /iii $P = Q$

(f) i / ii /iii $B \cap (A - B)$

(g) i / ii /iii $P \cup Q$

(h) i / ii /iii $A + B$

(i) i / ii /iii $(\exists x \in P)[x \in Q]$

(j) i / ii /iii $A \wedge B$