Math 220 – Homework 8

Due Friday 04/03 at the beginning of class

Total points: 171

PART A

Problems from the textbook:

- Section 4.3

problem	1(a)	1(b)*	2(a)	4(a)	5(a)
points	8	16	8	8	8

PART B

1. [30 points] Let $i \in \mathbb{Z}$ and $A_i = \{i-1, i+1\}$. Write the following sets in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

   (a) $\bigcup_{i=1}^{5} A_{2i}$

   (b) $\bigcup_{i=1}^{250} A_{2i}$

   (c) $\bigcup_{i=1}^{5} (A_i \cap A_{i+1})$

   (d) $\bigcup_{i=1}^{250} (A_i \cap A_{i+1})$

   (e) $\bigcup_{i=1}^{5} (A_{2i-1} \cap A_{2i+1})$

   (f) $\bigcup_{i=1}^{250} (A_{2i-1} \cap A_{2i+1})$

2. [15 points] Repeat the previous problem for $A_i = [i-1, i+1]$.

3. [15 points] Given $I = \{1, 2, 3, \ldots, 2018\}$. For each $i \in I$ define $B_i = \{i, i+1\}$. Write the following in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

   (a) $\bigcap_{i \in I} B_i$

   (b) $\bigcap_{i=j}^{j+1} B_i$

   (c) If $1 \leq j \leq k \leq 2018$, then $\bigcup_{i=j}^{k} B_i =$

4. Let $A = \{x, y, z, u, v\}$, $B = \{a, b, c, d\}$, and $C = \{5, 6, 7, 8, 9\}$.

   (a) [9 points] Write out three functions with domain A and codomain B making at least one of the functions have the property that its range coincides with its codomain (represent all functions by their graphs (see Definition 3 in notes)).

(b) [9 points] Write out two functions with domain B and codomain C(represent all functions by their graphs (see Definition 3 in notes)). Explain why you cannot define a function between these two sets for which the range equals its codomain.

5. Let $f : \mathbb{Z} \to \mathbb{R}$ and let $g : \mathbb{Z} \to \mathbb{R}$ be defined by $f(n) = \cos(\pi n)$ and $g(n) = (-1)^n$.

   (a) [5 points] Find $\text{ran}(f)$ and $\text{ran}(g)$ and represent your answers using roster notation. (No formal proofs are necessary).

   (b) [5 points] Find graphs G_f and G_g and show that $G_f = G_g$.

6. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2018 - 4x$. Prove that $\text{ran} f = \mathbb{R}$.

7. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^4$ and $S = \{y \in \mathbb{R} |\ y \geq 0\}$. Prove that $\text{ran} f = S$.

8. Let $X = \{x \in \mathbb{R} |\ x \neq -5\}$ and $f : X \to \mathbb{R}$ be defined by $f(x) = \dfrac{3x - 1}{x + 5}$.

   (a) [5 points] Determine the range of f.

   (b) * [10 points] Prove that your answer for $\text{ran} f$ is correct.