# Math 220-Homework 9

## Due Wednesday 11/11 at the beginning of class

## PART A

Problems from the textbook:

- Section 5.3 # 1b, 6(b,c,d,e (in part (e) state, prove or disprove the converse of parts 3&4 of Proposition 5.3.3 only))

## PART B

1. Let $a, b, c \in \mathbf{Z}$. Determine the truth or falsehood of the following statements.

   (a)  $\gcd(a, 0) = a$.

   (b)  Let $a$ and $b$ be not both zero. Then $\gcd(a, b) = \gcd(-a, b)$.

   (c)  The set $\mathbf{Z} - \mathbf{Z}^+$ of integers is closed with respect to multiplication.

   (d)  $0|b$ only if $b = 0$.

   (e)  If $a|c$ and $b|c$, then $ab|c$.

   (f)  If $a|b$ and $b|a$ then $a = b$.

2. Prove by induction that for every positive integer $n$ the following statements hold:

   (a) $2 + 6 + 10 + \ldots + (4n - 2) = 2n^2$.

   (b) $n^3 + 2n$ is divisible by $3$.  (Hint: $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$)

   (c) $\dfrac{1}{2 \cdot 3} + \dfrac{1}{3 \cdot 4} + \ldots + \dfrac{1}{(n + 1)(n + 2)} = \dfrac{n}{2(n + 2)}$.

   (d) $3|(2^{2n} - 1)$.

   (e) $5$ is a factor of $7^n - 2^n$.

3. (a) Use the Euclidean Algorithm to determine $\gcd(374, 946)$.

   (b) Find integers $x$ and $y$ such that $374x + 946y = \gcd(374, 946)$.