# Math 220 – Homework 9

## Due Thursday 11/08 at the beginning of class

Total points: 224

## PART A

Problems from the textbook:

- Section 4.3

| problem | 1(a) | 2(a) | 4(a) | 5(a) |
|---|---|---|---|---|
| points | 8 | 8 | 8 | 8 |

- Section 5.2

| problem | 1(a) | 1(b) | 2 |
|---|---|---|---|
| points | 8 | 10 | 10 |

## PART B

1. [10 points] For a real number $r$, define $S_r$ to be the interval $[r-1, r+2]$. Let $A = \{1, 3, 4\}$. Write the sets $\bigcup_{\alpha\in A} S_\alpha$ and $\bigcap_{\alpha\in A} S_\alpha$ in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

2. [10 points] Let $K = \{a, b, c\}$, $L = \{b, d, e\}$, $M = \{b, e, f\}$ and $S = \{K, L, M\}$. Write the sets $\bigcup_{X\in S} X$ and $\bigcap_{X\in S} X$ in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

3. [30 points] Let $i \in \mathbb{Z}$ and $A_i = \{i-1, i+1\}$. Write the following sets in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

   **(a)** $\bigcup_{i=1}^{5} A_{2i}$ **(b)** $\bigcup_{i=1}^{250} A_{2i}$ **(c)** $\bigcup_{i=1}^{5} (A_i \cap A_{i+1})$ **(d)** $\bigcup_{i=1}^{250} (A_i \cap A_{i+1})$ **(e)** $\bigcup_{i=1}^{5} (A_{2i-1} \cap A_{2i+1})$
   **(f)** $\bigcup_{i=1}^{250} (A_{2i-1} \cap A_{2i+1})$

4. [15 points] Repeat the previous problem for $A_i = [i-1, i+1]$.

5. [15 points] Given $I = \{1, 2, 3, \ldots, 2018\}$. For each $i \in I$ define $B_i = \{i, i+1\}$. Write the following in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

   **(a)** $\bigcap_{i\in I} B_i$ **(b)** $\bigcap_{i=j}^{j+1} B_i$ **(c)** $\bigcup_{i=j}^{k} B_i$, where $1 \le j < k \le 2018$

6. Let $A = \{x, y, z, u, v\}$, $B = \{a, b, c, d\}$, and $C = \{5, 6, 7, 8, 9\}$.

   (a) [9 points] Write out three functions with domain $A$ and codomain $B$ making at least one of the functions have the property that its range coincides with its codomain (represent all functions by their graphs (see Definition 3 in notes)).

   (b) [9 points] Write out two functions with domain $B$ and codomain $C$(represent all functions by their graphs (see Definition 3 in notes)). Explain why you cannot define a function between these two sets for which the range equals its codomain.

7. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2018 - 4x$. Prove that $\text{ran} f = \mathbb{R}$.

8. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^4$ and $S = \{y \in \mathbb{R} |\ y \geq 0\}$. Prove that $\text{ran} f = S$.

9. Let $X = \{x \in \mathbb{R} |\ x \neq -5\}$ and $f : X \to \mathbb{R}$ be defined by $f(x) = \dfrac{3x-1}{x+5}$.

   (a) [5 points] Determine the range of $f$.

   (b) * [10 points] Prove that your answer for $\text{ran} f$ is correct.

10. Express each of the following functions as a composition $f = g \circ h$. Be sure to give appropriate sets $A, B$, and $C$ such that $h : A \to B$ and $g : B \to C$. Note that neither $g$ nor $h$ should be an identity functions, but there may be many possible answers.

    (a) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sqrt[3]{e^{x^3} + 8}$

    (b) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \ln(x^2 + 1)$

    (c) [7 points] $f : \mathbb{Z} \to \mathbb{R}$ defined by $f(x) = \sin(\pi x + 1)$

11. * [10 points] Determine whether the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = \begin{cases} 2n, & \text{if } n \in \mathbb{E} \\ -n + 22, & \text{if } n \in \mathbb{O} \end{cases}$

    is surjective. Give a formal proof of your answer.