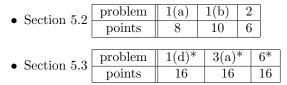
## Math 220 – Homework 9

## Due Tuesday 04/10 at the beginning of class

Total points: 157

## PART A

Problems from the textbook:



• Section 5.4  $\# 1(b)^*$  [24 points]

## PART B

- 1. Express each of the following functions as a composition  $f = g \circ h$ . Be sure to give appropriate sets A, B, and C such that  $h: A \to B$  and  $q: B \to C$ . Note that neither q nor h should be an identity functions, but there may be many possible answers.
  - (a) [7 points]  $f : \mathbb{R} \to \mathbb{R}$  defined by  $f(x) = \sqrt[3]{e^{x^3} + 8}$
  - (b) [7 points]  $f : \mathbb{R} \to \mathbb{R}$  defined by  $f(x) = \ln(x^2 + 1)$
  - (c) [7 points]  $f : \mathbb{Z} \to \mathbb{R}$  defined by  $f(x) = \sin(\pi x + 1)$
- 2. Let  $A = \{-1, 0, 1, 2, 3\}, B = \{a, b, c, d\}$ , and  $C = \{1, 2, 3, 4, 5\}$ . Using graph give examples of functions with the following properties. If such function does not exist, explain why.
  - (a) [3 points] an injective function with domain A and codomain B;
  - (b) [3 points] a surjective function with domain A and codomain B;
  - (c) [3 points] a surjective function with domain B and codomain C;
  - (d) [3 points] an injective function with domain C and codomain B;
  - (e) [3 points] an bijective function with domain A and codomain C:
  - (f) [3 points] an bijective function with domain A and codomain B.
- 3. \* Determine whether the following function is injection. Give a formal proof of your answer.
  - (a) [6 points]  $f : \mathbb{R} \to \mathbb{R}$  defined by  $f(x) = 16x^{16} 14x^{14} 14$ (b) [10 points]  $f : \mathbb{Z} \to \mathbb{Z}$  defined by  $f(n) = \begin{cases} n + 2018, & \text{if } n \in \mathbb{E} \\ -n + 2018, & \text{if } n \in \mathbb{O} \end{cases}$
- 4. \* [10 points] Determine whether the function  $f : \mathbb{Z} \to \mathbb{Z}$  defined by  $f(n) = \begin{cases} 2n, & \text{if } n \in \mathbb{E} \\ -n+22, & \text{if } n \in \mathbb{Q} \end{cases}$ is surjective.

Give a formal proof of your answer.

- 5. [10 points] The functions  $f, g: \mathbb{R} \to \mathbb{R}$  defined by f(x) = 2x + 1 and g(x) = 3x 5 are bijective. Determine the inverse function of  $g \circ f^{-1}$ .
- 6. [10 points] Let  $a, b \in \mathbb{R} \{0\}$  and let functions  $f, g : \mathbb{R} \to \mathbb{R}$  be defined by

$$f(x) = ax + b, \quad g(x) = x + \frac{b}{a}.$$

Compute the *inverse* function of  $g \circ f^{-1}$ .