Math 220 – Homework 9

Due Tuesday 04/10 at the beginning of class

Total points: 157

PART A

Problems from the textbook:

- Section 5.2

problem	1(a)	1(b)	2
points	8	10	6

- Section 5.3

problem	1(d)*	3(a)*	6*
points	16	16	16

- Section 5.4 # 1(b)* [24 points]

PART B

1. Express each of the following functions as a composition $f = g \circ h$. Be sure to give appropriate sets A, B, and C such that $h : A \to B$ and $g : B \to C$. Note that neither g nor h should be an identity functions, but there may be many possible answers.
   (a) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sqrt[3]{e^{x^3} + 8}$
   (b) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \ln(x^2 + 1)$
   (c) [7 points] $f : \mathbb{Z} \to \mathbb{R}$ defined by $f(x) = \sin(\pi x + 1)$
2. Let $A = \{-1, 0, 1, 2, 3\}$, $B = \{a, b, c, d\}$, and $C = \{1, 2, 3, 4, 5\}$. Using graph give examples of functions with the following properties. If such function does not exist, explain why.
   (a) [3 points] an injective function with domain A and codomain B;
   (b) [3 points] a surjective function with domain A and codomain B;
   (c) [3 points] a surjective function with domain B and codomain C;
   (d) [3 points] an injective function with domain C and codomain B;
   (e) [3 points] an bijective function with domain A and codomain C;
   (f) [3 points] an bijective function with domain A and codomain B.
3. * Determine whether the following function is injection. Give a formal proof of your answer.
   (a) [6 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 16x^{16} - 14x^{14} - 2x^2 + 1$
   (b) [10 points] $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = \begin{cases} n + 2018, & \text{if } n \in \mathbb{E} \\ -n + 2018, & \text{if } n \in \mathbb{O} \end{cases}$
4. * [10 points] Determine whether the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = \begin{cases} 2n, & \text{if } n \in \mathbb{E} \\ -n + 22, & \text{if } n \in \mathbb{O} \end{cases}$ is surjective.
   Give a formal proof of your answer.
5. [10 points] The functions $f, g : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 1$ and $g(x) = 3x - 5$ are bijective. Determine the inverse function of $g \circ f^{-1}$.
6. [10 points] Let $a, b \in \mathbb{R} - \{0\}$ and let functions $f, g : \mathbb{R} \to \mathbb{R}$ be defined by
   $$f(x) = ax + b, \quad g(x) = x + \frac{b}{a}.$$
   Compute the *inverse* function of $g \circ f^{-1}$.