Homework Assignment 4 in MATH 308 **due Feb 22, 2012**

Topics covered : *linear homogeneous equations of second order with constant coefficient: the cases of complex roots and repeated roots; method of reduction of order; nonhomogeneous equations and method of undetermined coefficients (corresponds to sections 3.3, 3.4, and 3.5 in the textbook).*

1. Use Euler's formula to write the given expression in the form $a + ib$:

   (a) $e^{\frac{3\pi}{4}i}$; (b) $e^{(4-\frac{\pi}{3}i)}$.

2. Consider the differential equation $y'' - 2y' + 10y = 0$.
   (a) Find the general solution of this equation;
   (b) Find the solution of the equation with the initial conditions $y(\frac{\pi}{2}) = 5$, $y'(\frac{\pi}{2}) = -1$. Sketch the graph of the solution and describe its behavior as $t \to -\infty$.

3. Consider the differential equation $y'' - 10y' + 25 = 0$.
   (a) Find the general solution of this equation;
   (b) Find the solution of this equation satisfying the initial conditions $y(0) = 3$, $y'(0) = \alpha$;
   (c) For the solutions obtained in the previous item find the values of α , if any, for which the solutions tends to $+\infty$ as $t \to +\infty$ and the values of α, if any, for which the solutions tend to $-\infty$ as $t \to +\infty$.

4. Given the solution $y_1(t) = t^{-1}$ of the differential equation $t^2y'' - 3ty' - 5y = 0, \quad t > 0$. Use the method of reduction of order to find a second solution $y_2(t)$ of this equation such that $\{y_1(t), y_2(t)\}$ is a fundamental set of solutions on $t > 0$.

5. Using the method of undetermined coefficients, find the general solution of the following differential equations:
   (a) $y'' + \omega_0^2 y = \sin \omega t$ (consider separately the case $\omega^2 \neq \omega_0^2$ and the case $\omega^2 = \omega_0^2$);
   (b) $y'' - 3y' + 2y = 5e^{2t} + e^{3t} \cos 2t$.

6. (**bonus**-20 points) Consider the differential equation $ay'' + by' + cy = 0$, where a, b, c are constant.
   (a) Prove that if the roots of the characteristic equation are real, then a solution of the differential equation is either everywhere zero or else can take on the value zero at most once.
   (b) If the roots of the characteristic equation are not real, what can you say about a number of time moments. where a solution of the differential equation takes on the value zero?
   (c) If a, b, c are positive constants, show that all solutions of the differential equation approach zero as $t \to +\infty$.