MATH 308, Spring 2013
EXAM 1 - VERSION A

LAST NAME (print)______________________________ FIRST NAME : ________________________

UIN: ____________________________________SECTION #: ________________ SEAT#: ________

DIRECTIONS:

- The use of a calculator is prohibited.
- The use of any electronic device is prohibited.
- In all problems present your solutions in the space provided.
- Be sure to read the instructions to each problem *carefully.*
- Use a pencil and be neat. If I can't read your answers, then I can't give you credit.
- *Show all your work* and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

1. Classify the following first-order ODE as separable, linear, both, or neither. If the equation is separable then separate variables (**do not solve, just separate the variables**).

   (a) [6pts] $y' - x\sqrt{y} = e^x y$

   (b) [6pts] $e^{y+\sec x}\mathrm{d}x - \mathrm{d}y = 0$

2. [11pts] A tank initially contains 200 gal of pure water. Then brine containing 5 lb/gal of salt is entering the tank with the rate 10 gal/min and then well stirred mixture is drained from the tank at the rate 10 gal/min. Write the initial values problem for the amount $Q(t)$ of the salt in tank at any time t. (**Do not solve ODE**).

3. [10pts] Determine (without solving the problem) an interval in which the solution of the given IVP is certain to exist:

$$(2t^2 + t - 1)y' + \sqrt{t}\, y = 13t^2, \quad y\left(\frac{1}{4}\right) = 2013$$

4. [12pts] Find general solution of the equation $ty' + 3y = t^2 - t + 1, \quad t > 0.$

5. [12pts] Determine whether the differential equation

$$(2y\sin x - e^x \sin y)\mathrm{d}x - (e^x \cos y + 2\cos x + 3y^2)\mathrm{d}y = 0$$

is exact. If it is exact, find the general solution.

6. (a) [12pts] Find the solution of the initial value problem

$$3y'' - 4y' + y = 0, \quad y(0) = \alpha, \quad y'(0) = \frac{1}{3} \tag{1}$$

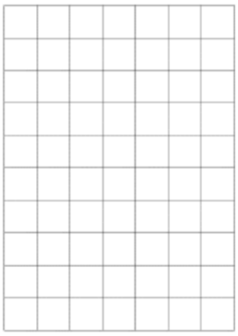
where α is a real parameter.

(b) [7pts] Determine all values of α, if any, for which the solution of the initial value problem (1) tends to $+\infty$ as $t \to +\infty$.

7. (a) [4pts] Find all equilibrium points of the differential equation:

$$y' = (y+2)(y^2-4) \tag{2}$$

(b) [8pts] Sketch the direction field for the equation (2) (use the grid below).



(c) [8pts] Let $y(t)$ be the solution of equation (2) satisfying the initial condition $y(0) = \alpha$. Based on the sketch of the direction field from the previous item, find the limit of $y(t)$ when $t \to \infty$ and when $t \to -\infty$ in each of the following cases (the value might be infinite). Fill the table:

$\alpha =$	2013	$-3/2$
$\lim_{t\to+\infty} y(t) =$		
$\lim_{t\to-\infty} y(t) =$		

(d) [4pts] Let $y(t)$ be the solution of equation (2) with $y(0) = -13$. Based on the sketch of the direction field decide whether $y(t)$ is monotonically decreasing or increasing and find to what value it approaches when t increases (the value might be infinite).

- End of Exam -

DO NOT WRITE BELOW!

# 1	# 2	# 3	# 4	# 5	# 6	# 7	Total
12	11	10	12	12	19	24	100