MATH 308, Spring 2013
EXAM 2 - VERSION B

LAST NAME (print)______________________________ FIRST NAME : ________________________

UIN: ____________________________________SECTION #: ________________ SEAT#: ________

DIRECTIONS:

- The use of a calculator is prohibited.
- The use of any electronic device is prohibited.
- In all problems present your solutions in the space provided.
- Be sure to read the instructions to each problem *carefully*.
- Use a pencil and be neat. If I can't read your answers, then I can't give you credit.
- *Show all your work* and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

DO NOT WRITE BELOW!

# 1	# 2	# 3	# 4	# 5	# 6	# 7	Total
28	16	10	12	14	11	9	100

1. A mass weighing $32\,lb$ stretches a spring 1.6ft. The mass is pushed *upward*, contracting the spring a distance 1/2 ft, and then set in motion with an *upward* velocity of $\sqrt{5}ft/s$. (Note: the gravitational acceleration $g = 32ft/s^2$)

   (a) [4pts] Write the initial value problem (the differential equation and the initial conditions) that describes the motion of the mass. Note that the positive direction of motion is assumed to be downward.

   (b) [6pts] Find the *position* u of the mass at any time t, if there is no damping and no external force.

   (c) [6pts] Determine the *natural frequency*, *period*, *amplitude* and *phase* of the motion.

-the problem is continued on the next page-

(d) [12pts] Now assume that mass is attached to a dashpot mechanism that has a damping constant of $1\,\frac{lb\cdot s}{ft}$ and is acted on by an external force of $2\sin(4t)$ lb. Determine the steady state solution of the obtained system.

2. Consider the differential equation $\frac{9}{4}y'' - 6y' + 4y = 0$.

   (a) [6pts] Find the general solution of this equation;

   (b) [4pts] Find the solution of this equation satisfying the initial conditions $y(0) = 1$, $y'(0) = \alpha$;

   (c) [6pts] For the solutions obtained in the previous item find the values of α , if any, for which the solutions tends to $+\infty$ as $t \to +\infty$ and the values of α, if any, for which the solutions tend to $-\infty$ as $t \to +\infty$.

3. [10pts] Given the solution $y_1(t) = t$ of the differential equation $2t^2y'' + 3ty' - 3y = 0, \quad t > 0$.
Use the *method of reduction of order* to find a second solution $y_2(t)$ of this equation such that $\{y_1(t), y_2(t)\}$ is a fundamental set of solutions on $t > 0$.

4. [12pts] The functions $y_1(t) = t + 1$ and $y_2(t) = e^t$ are known as fundamental set of $ty'' - (t + 1)y' + y = 0$. Use *the method of variation of parameter* to find the general solution of the equation $ty'' - (t+1)y' + y = t^2$.

5. For each of the following equations write down the form in which a particular solution should be found according to *the method of undetermined coefficients.* Justify your answer (**here you do not need to find the value of the undetermined coefficient/coefficients**):

   (a) [4pts] $y'' - 5y' - 14y = 2013e^{7t}$;

   (b) [4pts] $y'' - 6y' + 9y = (t - t^2)e^{3t}$;

   (c) [6pts] $y'' - 2y' + 17y = 3e^t \cos(4t) - e^{-t}$;

6. [11pts] Find *the inverse Laplace transform* of the function $F(s) = \dfrac{s-1}{(s-2)(s^2-2s+17)}$.

7. [9pts] Solve for $Y(s)$, the Laplace transform of the solution $y(t)$ to the given initial value problem (**you do not need to find the solution** $y(t)$ **itself here**). Do not simplify your answer.

$$y'' - 3y' + 25y = e^t(\sin(\sqrt{5}t) - t^4 + 5), \quad y(0) = 0, \quad y'(0) = 0.$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \|a\|$
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \|a\|$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$

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