MATH 308, Spring 2013
EXAM 3 (Sample Test)

LAST NAME (print)____________________________ FIRST NAME : ______________________

UIN: _________________________________SECTION #: ______________ SEAT#: _______

DIRECTIONS:

- The use of a calculator, laptop or computer is prohibited.
- In all problems present your solutions in the space provided.
- Be sure to read the instructions to each problem *carefully*.
- Use a pencil and be neat. If I can't read your answers, then I can't give you credit.
- *Show all your work* and *clearly indicate your final answer.* **Box it!** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

THE AGGIE CODE OF HONOR

centerline**"An Aggie does not lie, cheat or steal, or tolerate those who do."**

Signature: ____

Note: The following formulas may or may not be useful on this exam:

$$\sin A \cos B = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$$

$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

$$\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$$

1. [15pts] Find the Laplace transform of the function

$$f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 1 - 2t, & 1 \le t < 2 \\ t^2 + 1, & t \ge 2. \end{cases}$$

2. [8pts] Find the Laplace transform of the impulse function $f(t) = 7\,\delta\left(t - \frac{\pi}{4}\right)\sin t$

3. [10pts] Find the inverse Laplace transform of the function $F(s) = \dfrac{e^{-3s}(2s-5)}{s^2+8s+25}$.

4. [7pts] Given that $\left\{ \begin{pmatrix} 3e^{5t} \\ e^{5t} \end{pmatrix}, \begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix} \right\}$ is a fundamental set of the system $X' = AX$. Find all β_1 and β_2 such that if $X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the solution of the system $X' = AX$ with initial condition $X(0) = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$ then $X(t) \to 0$ as $t \to -\infty$.

5. [12pts] Find the general solution of the following system $\left\{ \begin{array}{rcl} x_1' & = & 2x_1 + 3x_2 \\ x_2' & = & 2x_1 + x_2 \end{array} \right.$

6. [8pts] Given a system of two linear differential equations $X' = AX$. It is known that the matrix A has two eigenvalues $\lambda_1 = 1 + 5i$ and $\lambda_2 = 1 - 5i$, and an eigenvector corresponding to λ_1 is $v_1 = \begin{pmatrix} 3 - i \\ 2 \end{pmatrix}$. Find the general **real** valued solution of this system.

7. [10pts] Find the inverse Laplace transform of $F(s) = \dfrac{s}{(s^2 + 49)^2}$, using the convolution theorem. (Evaluate the obtained convolution integral.)

8. [10pts] Consider the following system of differential equations $\begin{cases} x_1' = -2x_1 - 3x_2 + x_3 \\ x_2' = 5x_1 + 6x_2 - x_3 \\ x_3' = 3x_1 + x_2 \end{cases}$ It is known that $e^{2t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $e^{-t}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ are solutions of this system. It is also known that $\lambda = 3$ is an eigenvalue of the coefficient matrix and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ is a corresponding eigenvector. Based on these facts, find the solution of the given system satisfying the initial condition $x_1(0) = -3$, $x_2(0) = 4$, $x_3(0) = 7$.

9. [5pts] Transform the differential equation equation

$$y^{(4)} - t^3\, y^{(3)} - 2013e^t\, y'' - 2013y = t^2 \sin t$$

into a system of first order differential equations.

10. [10pts] Using the Laplace transform express the solution of the given initial value problem in terms of a convolution integral:

$$y'' + 16y = g(t), \quad y(0) = -1,\ y'(0) = 2. \tag{1}$$

11. [10pts] Consider the following system of differential equations $\begin{cases} x_1' = x_2 + x_3 \\ x_2' = x_1 + x_3 \\ x_3' = x_1 + x_2 \end{cases}$

It is known that $e^{2t}\begin{pmatrix}1\\1\\1\end{pmatrix}$ is a solution of this system and $\lambda = -1$ is an eigenvalue of multiplicity 2 of the coefficient matrix. Based on these facts, find the general solution of this system.

12. [15 pts] Use the *method of variation of parameter* to find the general solution of the system

$$\mathbf{X}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{X} + \begin{pmatrix} -\cos t \\ \sin t \end{pmatrix}$$

13. [10 pts] Find the general solution of the following system $\begin{cases} x_1' = -\frac{3}{2}x_1 + x_2 \\ x_2' = -\frac{1}{4}x_1 - \frac{1}{2}x_2 \end{cases}$