

- (a)  $\frac{1}{\pi}$  (b)  $\frac{2}{\pi}$  (c) 2 (d) 1 (e) 0

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2. Evaluate 
$$\int_0^1 xe^x dx$$
 (a)  $e$  (b)  $\frac{e}{2}$ 

- (c) 0
- (d) e 1 (e) 1

Title: Dec 4-7:52 PM (2 of 44)

3. Evaluate 
$$\int_0^1 \frac{1}{(x+1)(x+2)} dx$$

- (a)  $-\frac{1}{3}$  (b)  $\ln 2$  (c)  $\ln \left(\frac{4}{3}\right)$  (d) 3 (e)  $\tan^{-1}(2)$

Title: Dec 4-7:52 PM (3 of 44)

- 4. The region bounded by the curves y = x, y = 0, and x = 1 is rotated about the vertical line x = 2. Find the volume generated.
  - (a)  $2\pi$

- (b)  $\frac{8\pi}{3}$  (c)  $\frac{4\pi}{3}$  (d)  $\frac{7\pi}{8}$  (e)  $\frac{22\pi}{3}$

Title: Dec 4-7:53 PM (4 of 44)

5. Evaluate 
$$\int_3^8 \frac{3x}{\sqrt{x+1}} \, dx.$$

- (a) 32 (b)  $3\sqrt{8} 3\sqrt{3}$  (c) 7 (d) 62 (e) 12

Title: Dec 4-7:53 PM (5 of 44)

(a) 65	(b) 12	(c) 10	(d) 15	(e) 20

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7. Find the area bounded by the curves  $y = x^2 + 2$  and y = 3x from x = 0 to x = 2.

- (a) 1
- (b) 0
- (c)  $\frac{3}{2}$  (d)  $\frac{1}{3}$  (e)  $\frac{1}{6}$

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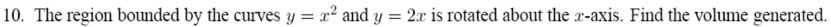
- 8. Evaluate  $\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx$  (a) 0 (b)  $\frac{2}{15}$  (c)  $\frac{4}{5}$  (d) 1 (e)  $\frac{5}{3}$

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9. Find 
$$\int \frac{1}{\sqrt{-3+4x-x^2}} dx$$

- (a)  $2\sqrt{-3+4x-x^2}+C$  (b)  $\sin^{-1}(-3+4x-x^2)+C$  (c)  $\tan^{-1}(x+2)+C$  (d)  $\sin^{-1}(x-2)+C$  (e)  $\tan^{-1}(x-1)+C$

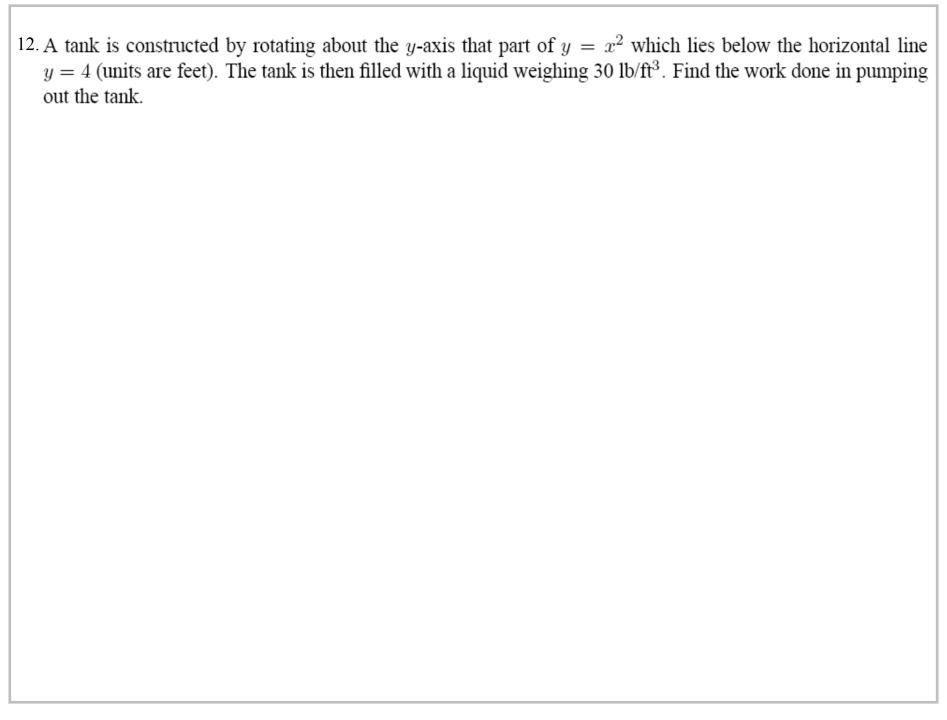
Title: Dec 4-7:54 PM (9 of 44)



- (a)  $12\pi$
- (b)  $\frac{64\pi}{15}$  (c)  $\frac{4}{3}\pi$  (d)  $2\pi$  (e)  $\frac{16\pi}{15}$

11. The triangle bounded by the straight lines $y = 0$ , $y = 4x$ and $y + 2x = 6$ is rotated about the x-axis. Set up but do not evaluate, integrals which give the volume generated using a) the disk/washer method, b) the cylindrical shells method.	p,

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13. Which of these integrals represents the arc length of 
$$y = x^3$$
 from  $x = 0$  to  $x = 1$ ?

a) 
$$\int_0^1 \sqrt{1+x^6} \, dx$$

b) 
$$\int_0^1 \sqrt{1+3x^2} \, dx$$

a) 
$$\int_0^1 \sqrt{1+x^6} \, dx$$
 b)  $\int_0^1 \sqrt{1+3x^2} \, dx$  c)  $\int_0^1 \sqrt{1+9x^4} \, dx$ 

d) 
$$\int_0^1 2\pi x^3 \sqrt{1+9x^4} \, dx$$

e) 
$$\int_0^1 \sqrt{1+x^3} \, dx$$

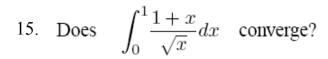
Title: Dec 4-7:56 PM (13 of 44)

14. By comparing the functions  $\frac{1}{1+x^3}$  and  $\frac{1}{x^3}$ , what conclusion can be drawn about  $\int_1^\infty \frac{1}{1+x^3} dx$ 

- a) No conclusion is possible b) It converges c) It does not converge
- d) Its value is 1/2

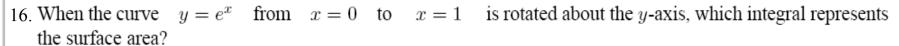
- e) Its value is 1

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- a) YES b) NO

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a) 
$$\int_0^1 2\pi \sqrt{1 + e^{2x}} \, dx$$
 b)  $\int_0^1 2\pi e^x \sqrt{1 + e^{2x}} \, dx$  c)  $\int_0^1 2\pi x \sqrt{1 + e^x} \, dx$ 

b) 
$$\int_0^1 2\pi e^x \sqrt{1 + e^{2x}} \, dx$$

c) 
$$\int_0^1 2\pi x \sqrt{1 + e^x} \, dx$$

d) 
$$\int_0^1 2\pi e^x \sqrt{1 + e^x} \, dx$$

e) 
$$\int_0^1 2\pi x \sqrt{1 + e^{2x}} \, dx$$

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17. Which of the statements about convergence of 
$$\sum_{n=1}^{\infty} a_n$$
,  $a_n \ge 0$ , are true?

- (1) If  $\lim_{n\to\infty} a_n = 0$  then the series converges,
- (2) If  $a_n \ge \frac{1}{n^2}$  then the series converges,
- (3) If  $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$  then the series diverges,
- (4) If  $a_n \leq \frac{1}{n}$  then the series converges.
  - (a) (4) only
  - (b) All
  - (c) None
  - (d) (1) only
  - (e) (2) and (3) only

None are true.(c)

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18. The interval of convergence of 
$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$$
 is

- (a) (-1,1]
- (b) [-1,1]
- (c)  $(-\infty, \infty)$
- (d) [-1,1)
- (e) (-1,1)

Title: Dec 4-8:00 PM (18 of 44)

19. Find  $\lim_{x\to 0} \frac{\cos(x^3)-1}{\sin(x^2)-x^2}$  (Maclaurin series are useful here)

- (a) 3
- (b) -1
- (c) 0
- (d) 1
- (e) 2

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20. The series 
$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

- (a) Diverges because  $a_n \to \infty$
- (b) Converges by the ratio test
- (c) Diverges by the ratio test
- (d) Diverges by the comparison test
- (e) Diverges by the integral test

Title: Dec 4-8:02 PM (20 of 44)

21. The area bounded by the curves y = 2x and  $y = \sqrt{x}$  is

(A) 
$$\pi \int_0^4 (\sqrt{x} - 2x) \, dx$$

(B) 
$$\int_0^{1/4} (\sqrt{x} - 2x) \, dx$$

(C) 
$$\pi \int_0^1 (2x - \sqrt{x})^2 dx$$

(D) 
$$\int_0^4 (2x - \sqrt{x}) \, dx$$

(E) 
$$2\pi \int_0^4 (2x - \sqrt{x}) x \, dx$$

Title: Dec 4-8:03 PM (21 of 44)

22. A trigonometric substitution converts the integral  $\int \sqrt{x^2 + 2x - 8} \, dx$  to

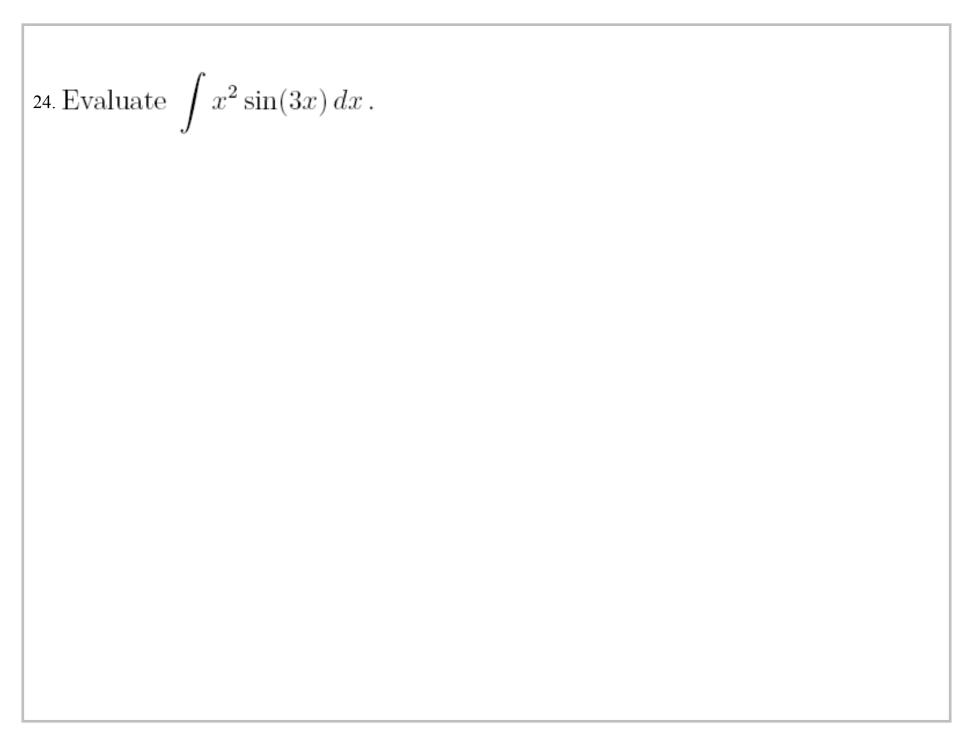
- (A)  $3 \int \tan^3 \theta \, d\theta$
- (B)  $9 \int \tan^2 \theta \sec \theta \, d\theta$
- (C)  $9 \int \sin^3 \theta \, d\theta$
- (D)  $3 \int \sin^2 \theta \cos \theta \, d\theta$
- (E)  $\int \tan \theta \sec^2 \theta \, d\theta$

Title: Dec 4-8:03 PM (22 of 44)

23. Find the average value of the function  $f(x) = \cos^3 x$  on the interval  $\left[0, \frac{\pi}{2}\right]$ .

- A)  $\frac{4}{3\pi}$
- (B)  $\frac{3}{\pi}$
- (C)  $\frac{1}{2}$
- $D) \frac{\pi}{2}$
- (E)  $\frac{1}{3}$

Title: Dec 4-8:04 PM (23 of 44)



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25. The improper integral  $\int_2^\infty \frac{2 + \cos x}{x^4} dx$ 

(A) diverges to  $+\infty$ .

(B) diverges, but does not approach  $\infty$  because the integrand oscillates.

(C) converges, by comparison with the integral  $\int_2^\infty \frac{3}{x^4} dx$ .

(D) converges to the value  $\frac{1}{12}$ .

(E) converges, because the integrand oscillates.

26. What integral represents the arc length of the parametric curve segment

$$x = 1 + \cos(2t), \quad y = t - \sin(2t), \quad 0 \le t \le \pi$$
?

(A) 
$$\int_0^{\pi} \sqrt{2 - 4\cos(2t) + 4\cos^2(2t)} dt$$

(B) 
$$\int_0^{\pi} \sqrt{6 - 4\cos(2t)} dt$$

(C) 
$$\int_0^{\pi} \sqrt{2 + 2\cos(2t) + t^2 - 2\sin(2t)} dt$$

(D) 
$$\int_0^{\pi} \sqrt{5 - 4\cos(2t)} dt$$

(E) 
$$\int_0^{\pi} \sqrt{3 + t^2 + 2\cos(2t) - 2\sin(2t)} dt$$

Title: Dec 4-8:05 PM (26 of 44)

27. The integral  $\int_0^\infty \frac{dx}{(x-2)^2}$ 

- (A) diverges, because of the behavior of the integrand at infinity.
- (B) diverges, because of the behavior of the integrand at zero.
- (C) converges, by comparison with the integral  $\int_1^\infty \frac{dx}{x^2}$ .
- (D) converges, because the integrand approaches a finite constant as  $x \to 0$ .
- (E) none of these.

Title: Dec 4-8:06 PM (27 of 44)

28. Evaluate  $\int \frac{x^2 + 1}{x^3 + 2x^2 + x} dx.$ 

Title: Dec 4-8:06 PM (28 of 44)

29. The surface having the equation  $x^2 + y^2 + z^2 + 4x - 2z = 20$  is

- A) a sphere with center at (-2,0,1) and radius 5.
- B) a sphere with center at (4,0,-2) and radius 4.
- C) a sphere with center at (2,0,-1) and radius  $\sqrt{20}$ .
- D) a sphere with center at (-4,0,2) and radius  $\sqrt{20}$ .
- E) not a sphere at all.

Title: Dec 4-8:07 PM (29 of 44)

30. Compute  $\int_{-1}^{1} \frac{1}{x^6} dx$ .

- **a**. 0
- **b**.  $\frac{2}{5}$
- **c**.  $-\frac{2}{7}$
- **d**.  $-\frac{2}{5}$
- e. Divergent

Title: Dec 4-8:10 PM (30 of 44)

31. Which of the following series are convergent?

(i) 
$$\sum_{n=1}^{\infty} \frac{100^n}{n!}$$

(i) 
$$\sum_{n=1}^{\infty} \frac{100^n}{n!}$$
 (ii)  $\sum_{n=1}^{\infty} \frac{2^n}{n+3^n}$ 

- a. both (i) and (ii)
- b. (i) only
- c. (ii) only
- d. neither

Title: Dec 4-8:11 PM (31 of 44)

32. Compute 
$$\sum_{n=0}^{\infty} \frac{5^{n+1}}{4^n}$$
.

- **a**. -20 **b**.  $\frac{20}{9}$
- **c**. 20
- **d**. 25
- e. divergent

Title: Dec 4-8:11 PM (32 of 44)

33. The series 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

- a. is absolutely convergent.
- **b**. is convergent but not absolutely convergent.
- **c**. is divergent to  $+\infty$ .
- **d**. is divergent to  $-\infty$ .
- e. is divergent but not to  $\pm \infty$ .

Title: Dec 4-8:12 PM (33 of 44)

34. Find the values of x such that the vectors (x,-1,3) and (2,-5,x) are orthogonal.

- $\mathbf{a}$ . -1 only
- **b**. 0 only
- c. 1 only
- $\mathbf{d}$ . 0 and 1 only
- e. 1 and -1 only

Title: Dec 4-8:12 PM (34 of 44)

35. Find the Taylor series for  $f(x) = x^2 + 3$  about x = 2.

**a**. 
$$7 + 4(x-2) + (x-2)^2$$

**b.** 
$$7 + 4(x-2) + 2(x-2)^2 + 4(x-2)^3$$

**c.** 
$$7 + 4(x-2) + (x-2)^2 + \frac{2}{3}(x-2)^4 + \cdots$$

**d**. 
$$7 + 4(x-2) + 2(x-2)^2 + 4(x-2)^3 + 2(x-2)^4 + \cdots$$

**e.** 
$$7 + 4(x-2) + 2(x-2)^2 + \frac{2}{3}(x-2)^3 + \frac{4}{3}(x-2)^4$$

Title: Dec 4-8:13 PM (35 of 44)

36. Find a power series centered at x = 0 for the function  $f(x) = \frac{x}{1 - 8x^3}$ , and determine its radius of convergence.

of convergence.

a. 
$$\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1}$$
  $R = \frac{1}{8}$ 

b.  $\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1}$   $R = 8$ 

c.  $\sum_{n=0}^{\infty} \frac{8^n}{n!} x^{3n+1}$   $R = 2$ 

d.  $\sum_{n=0}^{\infty} 8^n x^{3n+1}$   $R = \frac{1}{8}$ 

e.  $\sum_{n=0}^{\infty} 8^n x^{3n+1}$   $R = \frac{1}{2}$ 

**b.** 
$$\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1}$$
  $R = 8$ 

**c.** 
$$\sum_{n=0}^{\infty} \frac{8^n}{n!} x^{3n+1}$$
  $R = 2$ 

**d.** 
$$\sum_{n=0}^{\infty} 8^n x^{3n+1}$$
  $R = \frac{1}{8}$ 

**e.** 
$$\sum_{n=0}^{\infty} 8^n x^{3n+1}$$
  $R = \frac{1}{2}$ 

Title: Dec 4-8:13 PM (36 of 44)

37. Find the angle between the vectors  $\vec{u} = \langle 1, 1, 0 \rangle$  and  $\vec{v} = \langle 1, 2, 1 \rangle$ .

- $\mathbf{a}.~0^{\circ}$
- **b**. 30°
- **c**. 45°
- **d**. 60°
- **e**. 90°

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38. Evaluate the integral  $\int_0^{1/2} \frac{1}{1+x^3} dx$  as an infinite series.

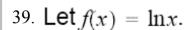
**a.** 
$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{2}\right)^{3n} = 1 - \frac{1}{2^3} + \frac{1}{2^6} - \frac{1}{2^9} + \cdots$$

**b.** 
$$\sum_{n=0}^{\infty} \frac{1}{3n+1} \left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} + \frac{1}{4 \cdot 2^4} + \frac{1}{7 \cdot 2^7} + \frac{1}{10 \cdot 2^{10}} + \cdots$$

c. 
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1} \left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} - \frac{1}{4 \cdot 2^4} + \frac{1}{7 \cdot 2^7} - \frac{1}{10 \cdot 2^{10}} + \cdots$$

**d.** 
$$\sum_{n=0}^{\infty} (-1)^n (3n-1) \left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{2}{2^2} + \frac{5}{2^5} - \frac{8}{2^8} + \cdots$$

e. 
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3n-1} \left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{1}{2 \cdot 2^2} + \frac{1}{5 \cdot 2^5} - \frac{1}{8 \cdot 2^8} + \cdots$$



Find the 3<sup>rd</sup> degree Taylor polynomial  $T_3$  for f(x) about x = 2.

**b**. If this polynomial  $T_3$  is used to approximate f(x) on the interval  $1 \le x \le 3$ , estimate the maximum error  $|R_3|$  in this approximation using Taylor's Inequality.

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## 40. Consider the points

$$P = (1,0,-1), \quad Q = (2,3,1) \quad \text{and} \quad R = (0,4,1)$$

Find a vector orthogonal to the plane determined by P, Q and R.

Find the area of the triangle with vertices P, Q and R.

4.1. Which statement most accurately describes the convergence or divergence of  $\int_{1}^{\infty} \frac{x \, dx}{\sqrt{x^5 + 1}}$ ?

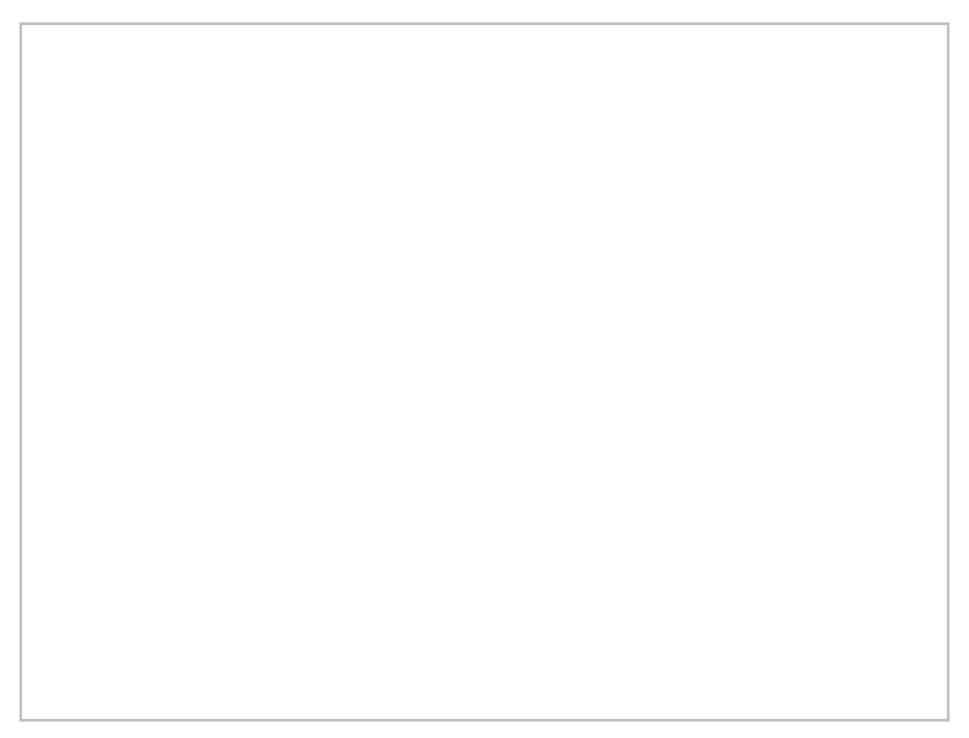
- a) The integral converges because  $\frac{x dx}{\sqrt{x^5 + 1}} \le \frac{1}{x^{5/2}}$  and  $\int_1^\infty \frac{dx}{x^{5/2}}$  converges.
- b) The integral diverges because  $\frac{x dx}{\sqrt{x^5 + 1}} \ge \frac{1}{x^{3/2}}$  and  $\int_1^\infty \frac{dx}{x^{3/2}} = \infty$ .
- c) The integral converges because  $\frac{x dx}{\sqrt{x^5 + 1}} \le \frac{1}{x^4}$  and  $\int_1^\infty \frac{dx}{x^4}$  converges.
- d) The integral diverges because  $\frac{x dx}{\sqrt{x^5 + 1}} \ge \frac{1}{x^4}$  and  $\int_1^\infty \frac{dx}{x^4} = \infty$ .
- e) The integral converges because  $\frac{x\,dx}{\sqrt{x^5+1}} \le \frac{1}{x^{3/2}}$  and  $\int_1^\infty \frac{dx}{x^{3/2}}$  converges.

- 42. Set up the integral that will compute the area of the surface obtained by revolving the curve  $x = (y 1)^2$  from (0, 1) to (1, 2)about the y-axis.
  - a)  $\int_{1}^{2} \sqrt{1 + 4(y 1)^{2}} \, dy$  b)  $\int_{0}^{1} 2\pi (x 1)^{2} \sqrt{1 + 4(x 1)^{2}} \, dx$  c)  $\int_{0}^{1} \sqrt{1 + 4(x 1)^{2}} \, dx$
  - d)  $\int_{1}^{2} 2\pi (y-1)^{2} \sqrt{1+4(y-1)^{2}} \, dy$  e)  $\int_{1}^{2} \pi (y-1)^{4} \, dy$

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43. If **A** and **B** are vectors in  $\mathbb{R}^3$ , which one of the following expressions has no meaning?

a)  $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$  b)  $(\mathbf{A} \cdot \mathbf{B}) \times \mathbf{C}$  c)  $(\mathbf{A} \cdot \mathbf{B}) \mathbf{C}$  d)  $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$  e)  $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ 



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