1. Find the average value of $f(x) = \sin x$ on the interval $[0, \pi]$.

(a) $\frac{1}{\pi}$ (b) $\frac{2}{\pi}$ (c) 2 (d) 1 (e) 0

2. Evaluate $\int_0^1 xe^x \, dx$

(a) e (b) $\frac{e}{2}$ (c) 0 (d) $e - 1$ (e) 1

3. Evaluate $\int_0^1 \frac{1}{(x+1)(x+2)}dx$

(a) $-\frac{1}{3}$ (b) $\ln 2$ (c) $\ln\left(\frac{4}{3}\right)$ (d) 3 (e) $\tan^{-1}(2)$

4. The region bounded by the curves $y = x$, $y = 0$, and $x = 1$ is rotated about the vertical line $x = 2$. Find the volume generated.

(a) 2π (b) $\frac{8\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{7\pi}{8}$ (e) $\frac{22\pi}{3}$

5. Evaluate $\int_3^8 \frac{3x}{\sqrt{x+1}}\, dx$.

(a) 32 (b) $3\sqrt{8} - 3\sqrt{3}$ (c) 7 (d) 62 (e) 12

6. When a spring of natural length 5 m. is extended to 6 m., the force required to hold it in position is 10N. Find the work done (in Joules) when the spring is extended from 6 m. long to 7 m. long.

(a) 65 (b) 12 (c) 10 (d) 15 (e) 20

7. Find the area bounded by the curves $y = x^2 + 2$ and $y = 3x$ from $x = 0$ to $x = 2$.

(a) 1 (b) 0 (c) $\frac{3}{2}$ (d) $\frac{1}{3}$ (e) $\frac{1}{6}$

8. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx$

(a) 0 (b) $\frac{2}{15}$ (c) $\frac{4}{5}$ (d) 1 (e) $\frac{5}{3}$

9. Find $\int \frac{1}{\sqrt{-3+4x-x^2}}dx$

(a) $2\sqrt{-3+4x-x^2}+C$ (b) $\sin^{-1}(-3+4x-x^2)+C$ (c) $\tan^{-1}(x+2)+C$

(d) $\sin^{-1}(x-2)+C$ (e) $\tan^{-1}(x-1)+C$

10. The region bounded by the curves $y = x^2$ and $y = 2x$ is rotated about the x-axis. Find the volume generated.

(a) 12π (b) $\frac{64\pi}{15}$ (c) $\frac{4}{3}\pi$ (d) 2π (e) $\frac{16\pi}{15}$

11. The triangle bounded by the straight lines $y = 0$, $y = 4x$ and $y + 2x = 6$ is rotated about the x-axis. Set up, but do not evaluate, integrals which give the volume generated using
a) the disk/washer method, b) the cylindrical shells method.

12. A tank is constructed by rotating about the y-axis that part of $y = x^2$ which lies below the horizontal line $y = 4$ (units are feet). The tank is then filled with a liquid weighing 30 lb/ft^3. Find the work done in pumping out the tank.

13. Which of these integrals represents the arc length of $y = x^3$ from $x = 0$ to $x = 1$?

a) $\int_0^1 \sqrt{1 + x^6}\, dx$ b) $\int_0^1 \sqrt{1 + 3x^2}\, dx$ c) $\int_0^1 \sqrt{1 + 9x^4}\, dx$

d) $\int_0^1 2\pi x^3 \sqrt{1 + 9x^4}\, dx$ e) $\int_0^1 \sqrt{1 + x^3}\, dx$

14. By comparing the functions $\dfrac{1}{1+x^3}$ and $\dfrac{1}{x^3}$, what conclusion can be drawn about $\displaystyle\int_1^\infty \frac{1}{1+x^3}dx$

a) No conclusion is possible **b)** It converges c) It does not converge
d) Its value is 1/2 e) Its value is 1

15. Does $\int_0^1 \frac{1+x}{\sqrt{x}} dx$ converge?

a) YES   b) NO

16. When the curve $y = e^x$ from $x = 0$ to $x = 1$ is rotated about the y-axis, which integral represents the surface area?

a) $\int_0^1 2\pi\sqrt{1+e^{2x}}\,dx$

b) $\int_0^1 2\pi e^x\sqrt{1+e^{2x}}\,dx$

c) $\int_0^1 2\pi x\sqrt{1+e^x}\,dx$

d) $\int_0^1 2\pi e^x\sqrt{1+e^x}\,dx$

e) $\int_0^1 2\pi x\sqrt{1+e^{2x}}\,dx$

17. Which of the statements about convergence of $\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0,$ are true?

(1) If $\lim_{n\to\infty} a_n = 0$ then the series converges,

(2) If $a_n \geq \dfrac{1}{n^2}$ then the series converges,

(3) If $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} = 1$ then the series diverges,

(4) If $a_n \leq \dfrac{1}{n}$ then the series converges.

(a) (4) only

(b) All

(c) None

(d) (1) only

(e) (2) and (3) only

None are true.(c)

18. The interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$ is

(a) $(-1, 1]$

(b) $[-1, 1]$

(c) $(-\infty, \infty)$

(d) $[-1, 1)$

(e) $(-1, 1)$

19. Find $\lim_{x \to 0} \frac{\cos(x^3) - 1}{\sin(x^2) - x^2}$ (Maclaurin series are useful here)

(a) 3

(b) -1

(c) 0

(d) 1

(e) 2

20. The series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(a) Diverges because $a_n \to \infty$

(b) Converges by the ratio test

(c) Diverges by the ratio test

(d) Diverges by the comparison test

(e) Diverges by the integral test

21. The area bounded by the curves $y = 2x$ and $y = \sqrt{x}$ is

(A) $\pi \displaystyle\int_0^4 (\sqrt{x} - 2x)\, dx$

(B) $\displaystyle\int_0^{1/4} (\sqrt{x} - 2x)\, dx$

(C) $\pi \displaystyle\int_0^1 (2x - \sqrt{x})^2\, dx$

(D) $\displaystyle\int_0^4 (2x - \sqrt{x})\, dx$

(E) $2\pi \displaystyle\int_0^4 (2x - \sqrt{x})\, x\, dx$

22. A trigonometric substitution converts the integral $\int \sqrt{x^2+2x-8}\,dx$ to

(A) $3\int \tan^3\theta\,d\theta$

(B) $9\int \tan^2\theta\sec\theta\,d\theta$

(C) $9\int \sin^3\theta\,d\theta$

(D) $3\int \sin^2\theta\cos\theta\,d\theta$

(E) $\int \tan\theta\sec^2\theta\,d\theta$

23. Find the average value of the function $f(x) = \cos^3 x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

A) $\frac{4}{3\pi}$

B) $\frac{3}{\pi}$

C) $\frac{1}{2}$

D) $\frac{\pi}{2}$

E) $\frac{1}{3}$

24. Evaluate $\int x^2 \sin(3x)\, dx$.

25. The improper integral $\int_2^\infty \frac{2+\cos x}{x^4}\,dx$

(A) diverges to $+\infty$.

(B) diverges, but does not approach ∞ because the integrand oscillates.

(C) converges, by comparison with the integral $\int_2^\infty \frac{3}{x^4}\,dx$.

(D) converges to the value $\frac{1}{12}$.

(E) converges, because the integrand oscillates.

26.What integral represents the arc length of the parametric curve segment

$$x = 1 + \cos(2t), \quad y = t - \sin(2t), \quad 0 \le t \le \pi\,?$$

(A) $\int_0^\pi \sqrt{2 - 4\cos(2t) + 4\cos^2(2t)}\,dt$

(B) $\int_0^\pi \sqrt{6 - 4\cos(2t)}\,dt$

(C) $\int_0^\pi \sqrt{2 + 2\cos(2t) + t^2 - 2\sin(2t)}\,dt$

(D) $\int_0^\pi \sqrt{5 - 4\cos(2t)}\,dt$

(E) $\int_0^\pi \sqrt{3 + t^2 + 2\cos(2t) - 2\sin(2t)}\,dt$

27. The integral $\int_0^\infty \frac{dx}{(x-2)^2}$

(A) diverges, because of the behavior of the integrand at infinity.

(B) diverges, because of the behavior of the integrand at zero.

(C) converges, by comparison with the integral $\int_1^\infty \frac{dx}{x^2}$.

(D) converges, because the integrand approaches a finite constant as $x \to 0$.

(E) none of these.

28. Evaluate $\displaystyle\int \frac{x^2+1}{x^3+2x^2+x}\,dx\,.$

29. The surface having the equation $x^2 + y^2 + z^2 + 4x - 2z = 20$ is

A) a sphere with center at $(-2, 0, 1)$ and radius 5.

B) a sphere with center at $(4, 0, -2)$ and radius 4.

C) a sphere with center at $(2, 0, -1)$ and radius $\sqrt{20}$.

D) a sphere with center at $(-4, 0, 2)$ and radius $\sqrt{20}$.

E) not a sphere at all.

30. Compute $\int_{-1}^{1} \frac{1}{x^6}\, dx$.

a. 0

b. $\frac{2}{5}$

c. $-\frac{2}{7}$

d. $-\frac{2}{5}$

e. Divergent

31. Which of the following series are convergent?

(i) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ (ii) $\sum_{n=1}^{\infty} \frac{2^n}{n+3^n}$

a. both (i) and (ii)
b. (i) only
c. (ii) only
d. neither

32. Compute $\sum_{n=0}^{\infty} \frac{5^{n+1}}{4^n}$.

a. -20

b. $\frac{20}{9}$

c. 20

d. 25

e. divergent

33. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

a. is absolutely convergent.

b. is convergent but not absolutely convergent.

c. is divergent to $+\infty$.

d. is divergent to $-\infty$.

e. is divergent but not to $\pm\infty$.

34. Find the values of x such that the vectors $\langle x, -1, 3 \rangle$ and $\langle 2, -5, x \rangle$ are orthogonal.

a. -1 only
b. 0 only
c. 1 only
d. 0 and 1 only
e. 1 and -1 only

35. Find the Taylor series for $f(x) = x^2 + 3$ about $x = 2$.

a. $7 + 4(x-2) + (x-2)^2$

b. $7 + 4(x-2) + 2(x-2)^2 + 4(x-2)^3$

c. $7 + 4(x-2) + (x-2)^2 + \frac{2}{3}(x-2)^4 + \cdots$

d. $7 + 4(x-2) + 2(x-2)^2 + 4(x-2)^3 + 2(x-2)^4 + \cdots$

e. $7 + 4(x-2) + 2(x-2)^2 + \frac{2}{3}(x-2)^3 + \frac{4}{3}(x-2)^4$

36. Find a power series centered at $x = 0$ for the function $f(x) = \dfrac{x}{1 - 8x^3}$, and determine its radius of convergence.

a. $\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1}$ $R = \dfrac{1}{8}$

b. $\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1}$ $R = 8$

c. $\sum_{n=0}^{\infty} \dfrac{8^n}{n!} x^{3n+1}$ $R = 2$

d. $\sum_{n=0}^{\infty} 8^n x^{3n+1}$ $R = \dfrac{1}{8}$

e. $\sum_{n=0}^{\infty} 8^n x^{3n+1}$ $R = \dfrac{1}{2}$

37. Find the angle between the vectors $\vec{u} = \langle 1, 1, 0 \rangle$ and $\vec{v} = \langle 1, 2, 1 \rangle$.

a. $0°$

b. $30°$

c. $45°$

d. $60°$

e. $90°$

38. Evaluate the integral $\int_0^{1/2} \frac{1}{1+x^3}\,dx$ as an infinite series.

a. $\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2}\right)^{3n} = 1 - \frac{1}{2^3} + \frac{1}{2^6} - \frac{1}{2^9} + \cdots$

b. $\sum_{n=0}^{\infty} \frac{1}{3n+1}\left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} + \frac{1}{4 \bullet 2^4} + \frac{1}{7 \bullet 2^7} + \frac{1}{10 \bullet 2^{10}} + \cdots$

c. $\sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1}\left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} - \frac{1}{4 \bullet 2^4} + \frac{1}{7 \bullet 2^7} - \frac{1}{10 \bullet 2^{10}} + \cdots$

d. $\sum_{n=0}^{\infty}(-1)^n(3n-1)\left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{2}{2^2} + \frac{5}{2^5} - \frac{8}{2^8} + \cdots$

e. $\sum_{n=0}^{\infty} \frac{(-1)^n}{3n-1}\left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{1}{2 \bullet 2^2} + \frac{1}{5 \bullet 2^5} - \frac{1}{8 \bullet 2^8} + \cdots$

39. Let $f(x) = \ln x$.

Find the 3^{rd} degree Taylor polynomial T_3 for $f(x)$ about $x = 2$.

b. If this polynomial T_3 is used to approximate $f(x)$ on the interval $1 \leq x \leq 3$, estimate the maximum error $|R_3|$ in this approximation using Taylor's Inequality.

40. Consider the points

$$P = (1, 0, -1), \quad Q = (2, 3, 1) \quad \text{and} \quad R = (0, 4, 1)$$

Find a vector orthogonal to the plane determined by P, Q and R.

Find the area of the triangle with vertices P, Q and R.

41. Which statement most accurately describes the convergence or divergence of $\int_1^\infty \frac{x\,dx}{\sqrt{x^5+1}}$?

a) The integral converges because $\frac{x\,dx}{\sqrt{x^5+1}} \leq \frac{1}{x^{5/2}}$ and $\int_1^\infty \frac{dx}{x^{5/2}}$ converges.

b) The integral diverges because $\frac{x\,dx}{\sqrt{x^5+1}} \geq \frac{1}{x^{3/2}}$ and $\int_1^\infty \frac{dx}{x^{3/2}} = \infty$.

c) The integral converges because $\frac{x\,dx}{\sqrt{x^5+1}} \leq \frac{1}{x^4}$ and $\int_1^\infty \frac{dx}{x^4}$ converges.

d) The integral diverges because $\frac{x\,dx}{\sqrt{x^5+1}} \geq \frac{1}{x^4}$ and $\int_1^\infty \frac{dx}{x^4} = \infty$.

e) The integral converges because $\frac{x\,dx}{\sqrt{x^5+1}} \leq \frac{1}{x^{3/2}}$ and $\int_1^\infty \frac{dx}{x^{3/2}}$ converges.

42. Set up the integral that will compute the area of the surface obtained by revolving the curve $x = (y-1)^2$ from $(0,1)$ to $(1,2)$about the y-axis.

a) $\int_1^2 \sqrt{1+4(y-1)^2}\,dy$ b) $\int_0^1 2\pi(x-1)^2\sqrt{1+4(x-1)^2}\,dx$ c) $\int_0^1 \sqrt{1+4(x-1)^2}\,dx$

d) $\int_1^2 2\pi(y-1)^2\sqrt{1+4(y-1)^2}\,dy$ e) $\int_1^2 \pi(y-1)^4\,dy$

43. If $\mathbf{A}$ and $\mathbf{B}$ are vectors in R^3, which one of the following expressions has no meaning?

a) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ b) $(\mathbf{A} \cdot \mathbf{B}) \times \mathbf{C}$ c) $(\mathbf{A} \cdot \mathbf{B})\mathbf{C}$ d) $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$ e) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$