Section 3.3:Rates Of Change In The Natural And Social Sciences.

Let $s(t)$ be the position function of an object. Its rate of change with respect to time is the velocity:

$$v(t) = s'(t).$$

- If $v(t) = 0$ then the object is at rest;
- if $v(t) > 0$ then the object is moving in the positive direction (i.e. is advancing, up or right);
- if $v(t) < 0$ then the object is moving in the negative direction (i.e. is retreating, down or left);

Rectilinear motion (motion along a line): A particle representing some object is allowed to move in either direction along a line.

EXAMPLE 1. *A particle is moving in a straight line. Its position is given by*

$$s(t) = 4t^3 - 9t^2 + 6t + 2,$$

where t is measured in seconds and s is measured in meters.

(a) *Find the velocity $v(t)$ of the particle at time t.*

$v(t) = s'(t) = 12t^2 - 18t + 6$

(b) *When is the particle at rest?*

Find t s.t. $v(t) = 0$

$12t^2 - 18t + 6 = 0 \quad (\times \frac{1}{6})$

$2t^2 - 3t + 1 = 0$

$(t-1)(t-\frac{1}{2}) = 0$

$t = 1$ or $t = \frac{1}{2}$

$t_{1,2} = \frac{3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2}$

$\Downarrow$

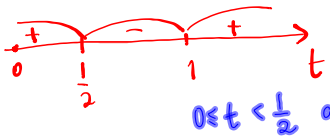
$t = 1$ or $t = \frac{1}{2}$

$ax^2 + bx + c = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

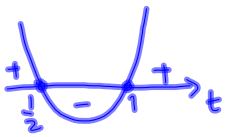
(c) *When is the particle moving in the positive direction?*

$v(t) = 6 \cdot 2(t-1)(t-\frac{1}{2}) > 0$



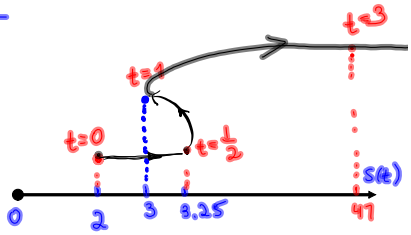
$0 < t < \frac{1}{2}$ and $t > 1$

2nd way



(d) *Draw a diagram to represent the motion of the particle.*

t	$s(t) = 4t^3 - 9t^2 + 6t + 2$
0	2
$\frac{1}{2}$	3.25
1	3
3	47



3

(e) *Find the total distance the particle traveled during the first three seconds.* (Hint:Calculate each distance between turns and then add to get the total.)

total distance $= |s(\frac{1}{2}) - s(0)| + |s(1) - s(\frac{1}{2})| + |s(3) - s(1)|$

$= |3.25 - 2| + |3 - 3.25| + |47 - 3| = 45.5$ m

EXAMPLE 2. *A ball is thrown vertically upward with ~~a velocity of 80ft/s.~~ Its height after t seconds is given by*

$$s(t) = 80t - 16t^2.$$

What is the maximum height reached by the ball? $\Longrightarrow$ $v'(t) = 0$

$v(t) = s'(t) = 80 - 32t$

$\Downarrow$

$80 - 32t = 0$

$80 = 32t$

$t = \frac{80}{32} = 2.5\ s$

$s(2.5) = 80 \cdot 2.5 - 16 \cdot 2.5^2 = 100\ ft$