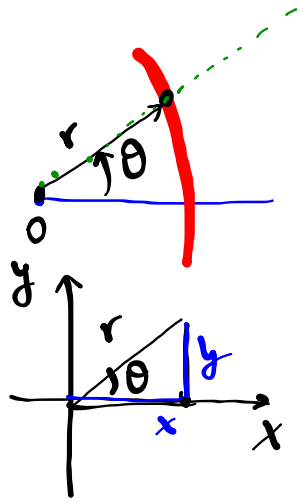
13.4: Polar Coordinates

REVIEW: $r \geq 0$, $0 \leq \theta \leq 2\pi$



The connection between polar and Cartesian coordinates:

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sin\theta = \frac{y}{r}$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$r^2 = x^2 + y^2$ $\qquad$ $\tan\theta = \frac{y}{x}$

REMARK 1. In converting from the Cartesian to polar coordinates we must choose θ so that the point (r, θ) lies in the correct quadrant.

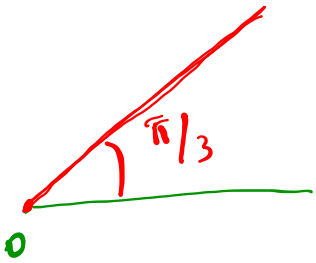
EXAMPLE 2. *What curve is represented by the polar equation*

(a) $r = 12$ $= \sqrt{x^2+y^2}$

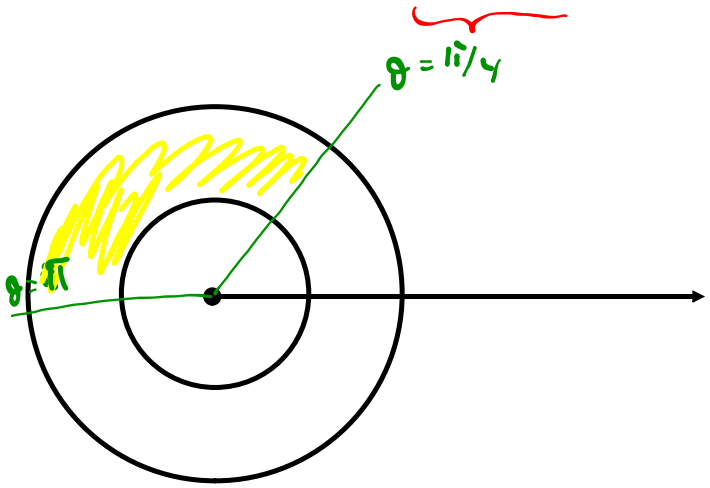
$x^2+y^2 = 12^2$ circle with radius 12 centered at origin.

(b) $\theta = \frac{\pi}{3}$

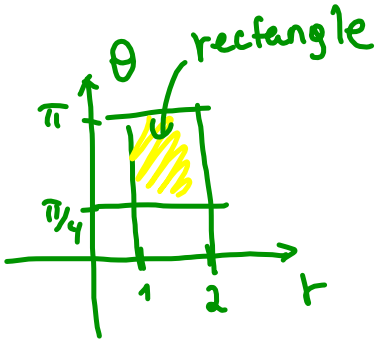
ray



EXAMPLE 3. *Sketch the region in the Cartesian plane consisting of points whose polar coordinates satisfy the following conditions:* $1 \leq r \leq 2, \quad \pi/4 \leq \theta \leq \pi.$



Note



EXAMPLE 4. *Find a polar equation for the curve represented by the given Cartesian equation:*

(a) $x^2 + y^2 = 2by$

Note that this is the circle $x^2 + (y-b)^2 = b^2$

$r^2 = 2br\sin\theta$

$$r = 2b\sin\theta$$

(b) $(x-a)^2 + y^2 = a^2$ (the same as $x^2 + y^2 = 2ax$)

$r^2 = 2ar\cos\theta$

$$r = 2a\cos\theta$$