Section 3.3:Rates Of Change In The Natural And Social Sciences.

Let $s(t)$ be the position function of an object. Its rate of change with respect to time is the velocity:

$$\boxed{v(t) = s'(t).}$$

- If $v(t) = 0$ then the object is at rest;
- if $v(t) > 0$ then the object is moving in the positive direction (i.e. is advancing, up or right);
- if $v(t) < 0$ then the object is moving in the negative direction (i.e. is retreating, down or left);

Rectilinear motion (motion along a line): A particle representing some object is allowed to move in either direction along a line.

EXAMPLE 1. *A particle is moving in a straight line. Its position is given by*

$$s(t) = 4t^3 - 9t^2 + 6t + 2,$$

where t is measured in seconds and s is measured in meters.

(a) *Find the velocity $v(t)$ of the particle at time t.*

$$V(t) = S'(t) = 12t^2 - 18t + 6 = 6(2t^2 - 3t + 1)$$

(b) *When is the particle at rest?*

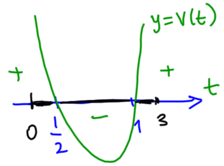
$$V(t) = 0$$

$$2t^2 - 3t + 1 = 0$$

$$2(t-1)(t-\tfrac{1}{2}) = 0$$

$$\boxed{t = 1\,s \quad \text{and} \quad t = \tfrac{1}{2}\,s}$$

(c) *When is the particle moving in the positive direction?* $V(t) > 0$



$(-\infty, \frac{1}{2}) \cup (1, \infty)$

For applications $(0, \frac{1}{2}) \cup (1, \infty)$.

(d) *Draw a diagram to represent the motion of the particle.*

t	$s(t)$
0	2
$\frac{1}{2}$	3.25
1	3
3	47

(e) *Find the total distance the particle traveled during the first three seconds.* (Hint:Calculate each distance between turns and then add to get the total.)

total distanse travelled $= |s(0) - s(\tfrac{1}{2})| + |s(\tfrac{1}{2}) - s(1)| + |s(1) - s(3)|$

$= |2 - 3.25| + |3.25 - 3| + |3 - 47| = 45.5\ m.$

EXAMPLE 2. *A ball is thrown vertically upward with a velocity of 80ft/s. Its height after t seconds is given by*

$$s(t) = 80t - 16t^2.$$

What is the maximum height reached by the ball?

$$V(t) = s'(t) = 80 - 32t$$

Ball reaches its max height when

$V(t) = 0$, i.e. $80 - 32t = 0$

$$t = \frac{80}{32} = \frac{5}{2} \text{ s.}$$

$$s\left(\frac{5}{2}\right) = 80 \cdot \frac{5}{2} - 16 \cdot \left(\frac{5}{2}\right)^2 = 40 \cdot 5 - 4 \cdot 25 = 200 - 100 = 100 \text{ ft}$$

EXAMPLE 3. *A spherical balloon is being inflated. Find the rate of increase of the volume with respect to the radius r when r is* 1ft. (Recall that the volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

$$V(r) = \frac{4}{3}\pi r^3$$

$$V'(r) = \frac{4}{3}\pi \cdot 3r^2 = \underline{4\pi r^2}$$