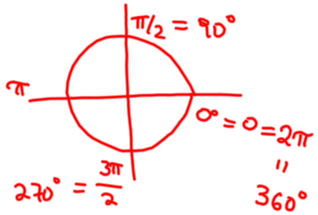
Angles. Angles can be measured in degrees or radians.

Everything in most calculus classes will be done in radians.

The angle given by a complete revolution contains 360^0, or 2π radians, i.e. $360^0 = 2\pi$ rad.

Hence,

$$\pi \text{ rad} = 180^0$$



and

$$\boxed{1\text{rad} = \left(\frac{180}{\pi}\right)^0, \quad 1^0 = \frac{\pi}{180}\text{rad.}}$$

EXAMPLE 1. *Convert* -315^0 *to radians.*

$$-315^\circ = -315 \cdot 1^\circ = -3\cancel{15}^{7} \cdot \frac{\pi}{\cancel{180}_{4}} \text{rad} = -\frac{7\pi}{4}\text{rad}$$

EXAMPLE 2. *Convert* $\dfrac{5\pi}{6}$ *to degrees.*

$$\frac{5\pi}{6}\text{rad} = \frac{5\pi}{6} \cdot 1\text{rad} = \frac{5\pi}{6} \cdot \left(\frac{180}{\pi}\right)^\circ = \left(\frac{5\cancel{\pi} \cdot \cancel{180}^{30}}{\cancel{6}_{1}\,\cancel{\pi}}\right)^\circ = 150^\circ$$

For basic angles we have the following table (Know it!)

Degree	0	30	45	60	90	180	270	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

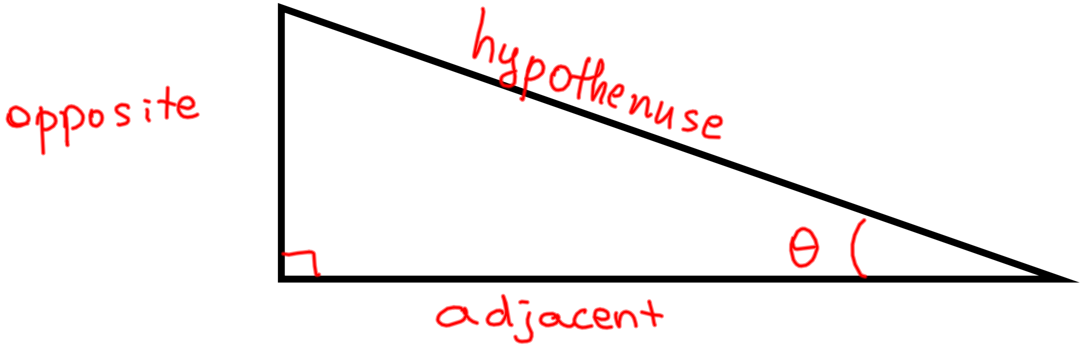
Six trigonometric functions and how they relate to each other.

$$\cos x \qquad \sin x$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

For an acute angle θ all the trig functions can be defined as ratios of sides of a right triangle:

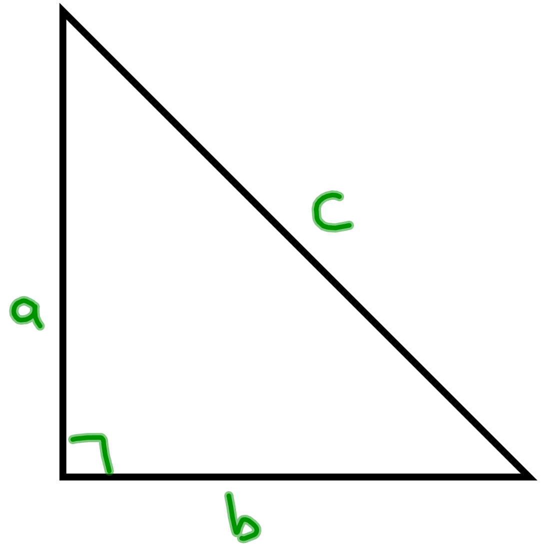


$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

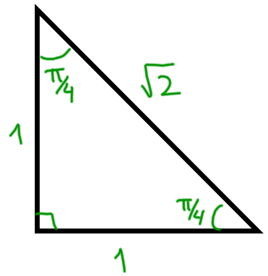
Pythagorean Theorem

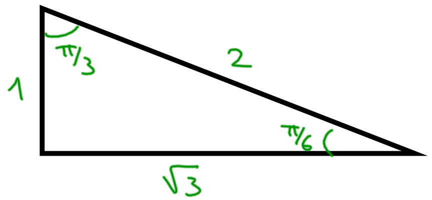


$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

The exact trigonometric ratios for certain angles can be read from the following two special triangles.





$$\frac{\text{opp}}{\text{hyp.}} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$

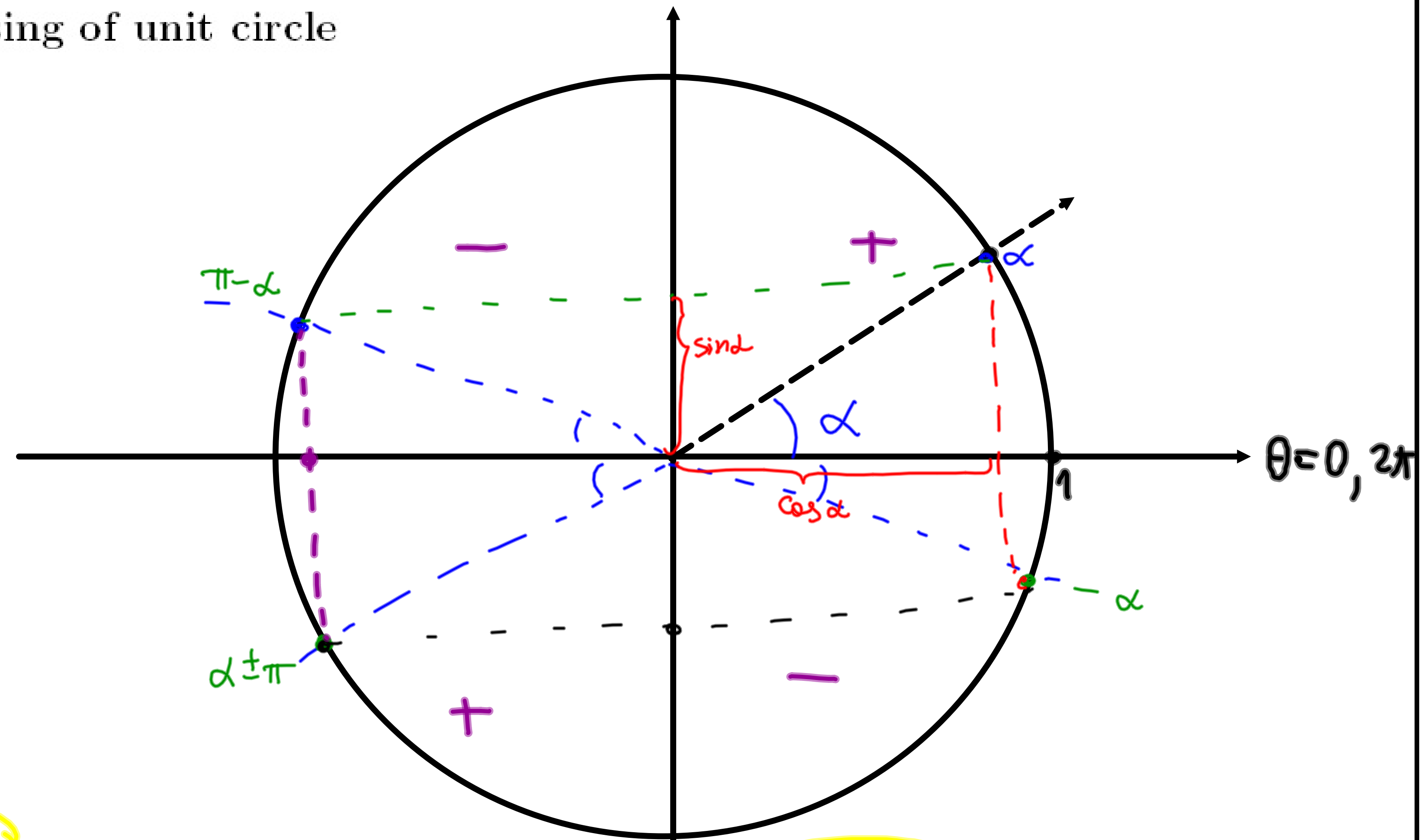
$$\tan\frac{\pi}{4} = 1 \qquad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	1	2	3	4
cos	4	3	2	1	0
tan					

/2

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Using of unit circle



$\sin\alpha = \sin(\pi-\alpha) = -\sin(\alpha\pm\pi) = -\sin(-\alpha)$ odd function

$\cos\alpha = \cos(-\alpha) = -\cos(\pi-\alpha) = -\cos(\alpha\pm\pi)$ even function

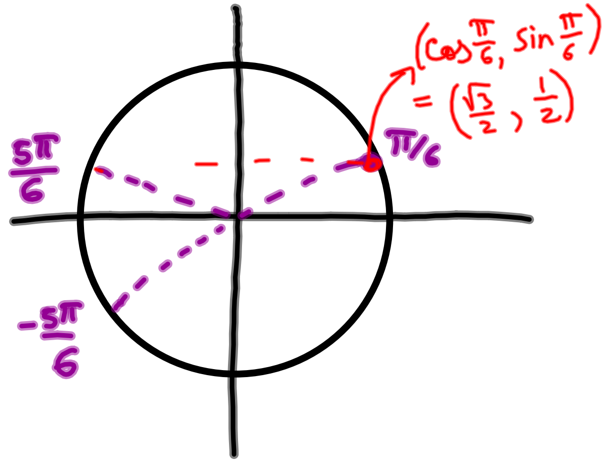
$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \tan(\alpha\pm\pi) = -\tan(\pi-\alpha) = -\tan(-\alpha)$

odd function

EXAMPLE 3. *Evaluate each of the following:*

(a) $\sin\frac{5\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

(b) $\sin\left(-\frac{5\pi}{6}\right) = -\sin\frac{5\pi}{6} = -\frac{1}{2}$



(c) $\cos\left(\frac{4\pi}{3}\right) = \cos\left(\frac{3\pi+\pi}{3}\right) = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$

(d) $\cos\left(-\frac{4\pi}{3}\right) = \cos\frac{4\pi}{3} = -\frac{1}{2}$

$\cos(-x) = \cos(x)$, i.e. cos is even function

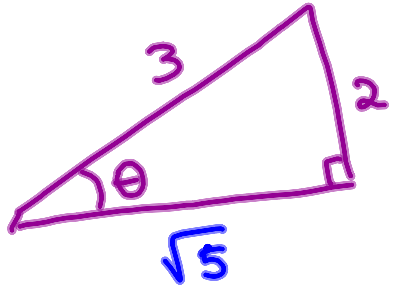
$\frac{\sin(-\frac{\pi}{4})}{\cos(-\frac{\pi}{4})} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$

(e) $\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

$\tan(-x) = -\tan x$, i.e. tan is odd function

EXAMPLE 4. *If* $\sin\theta = \dfrac{2}{3}$ *and* $0 < \theta < \pi/2$, *find the other five trigonometric functions of* θ.

θ is acute



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{3}$$

$$\text{adj} = \sqrt{\text{hyp}^2 - \text{opp}^2} = \sqrt{3^2 - 2^2} = \sqrt{5}$$

P.T.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{5}}{3}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{\sqrt{5}} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{5}}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{3}{\sqrt{5}} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{3}{2}$$

Know these identities

$$\Rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$$
$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1} \qquad \tan^2 + 1 = \sec^2\theta$$

$$\rightarrow \sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

EXAMPLE 5. *Find all values of x in the interval $[0, 2\pi]$ such that*

(a) $\sin 2x = \cos x$

$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ OR $2\sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Answer: $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\sin x = \frac{3}{2}$ no solution because $-1 \le \sin x \le 1$ for all x, and $\frac{3}{2} \notin [-1, 1]$

$-\frac{\pi}{2} < y < 0$

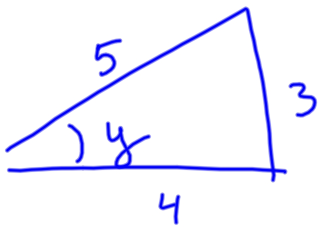
EXAMPLE 6. *If* $\sec y = \frac{5}{4}$, *where* $\cancel{0 < y < \pi/2}$, *evaluate* $\sin 2y$.

$$\sin 2y = 2 \sin y \cos y = 2 \cdot \frac{4}{5} \cdot \left(-\frac{3}{5}\right) = -\frac{24}{25}$$

$$\sec y = \frac{1}{\cos y} = \frac{5}{4} \Rightarrow \cos y = \frac{4}{5}$$

$$\Downarrow$$

$$\sin y = -\frac{3}{5}$$

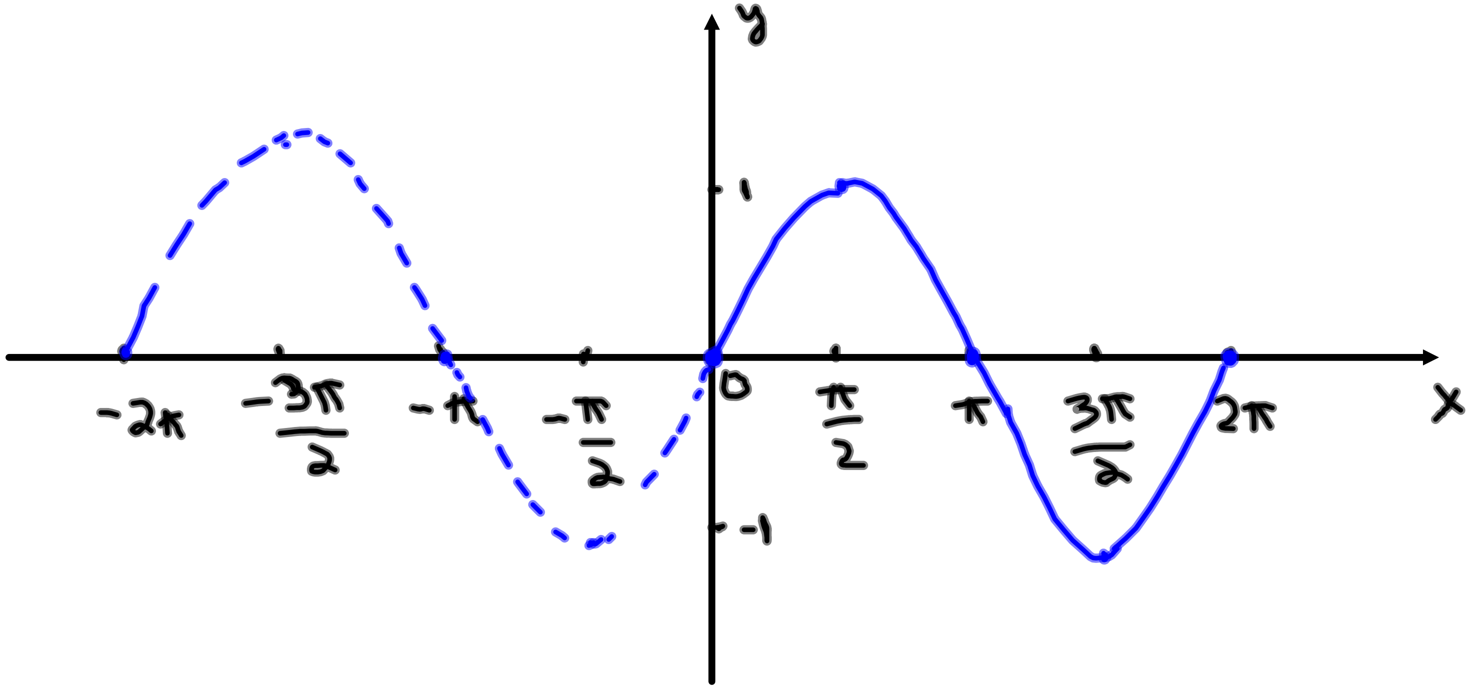


Another way:

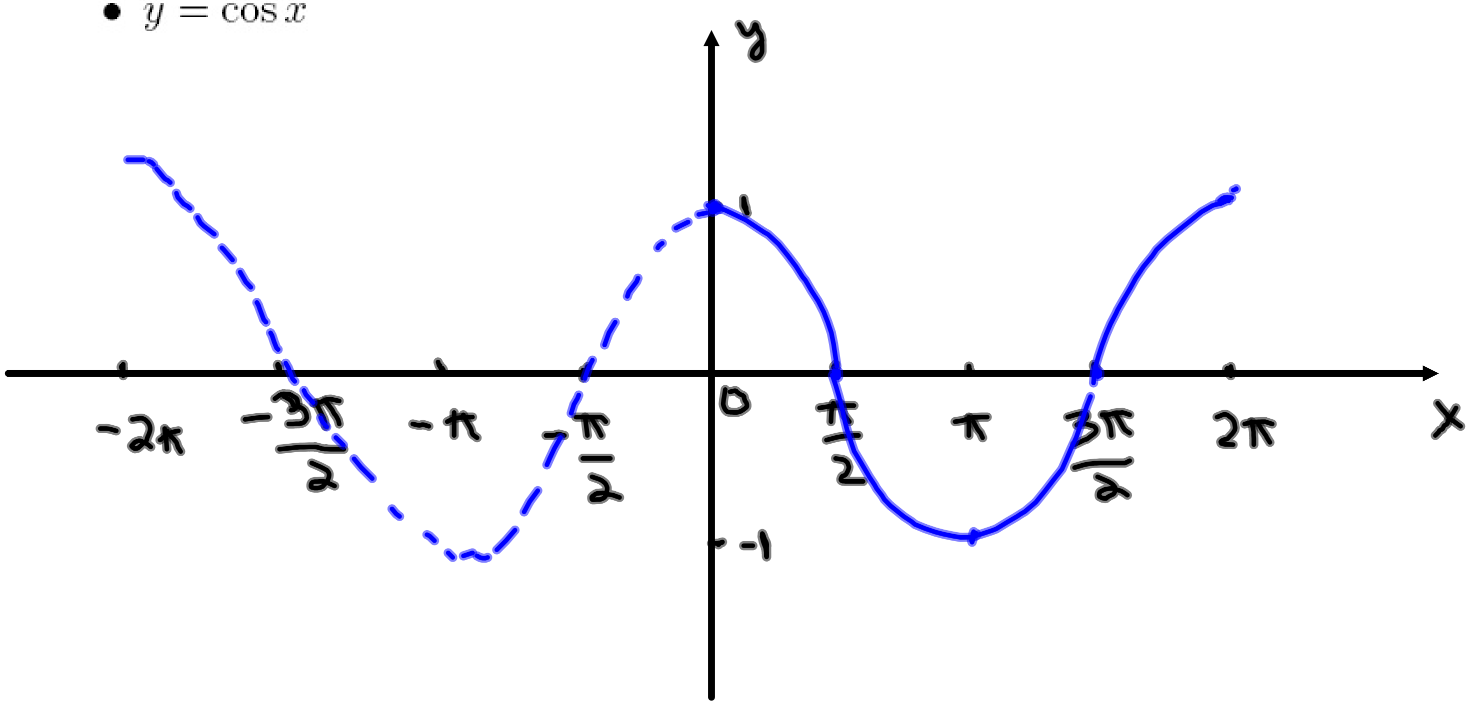
$$\sin y = \pm\sqrt{1 - \cos^2 y} = -\sqrt{1 - \left(\frac{4}{5}\right)^2} = -\frac{3}{5}$$

Graphs of the trig functions

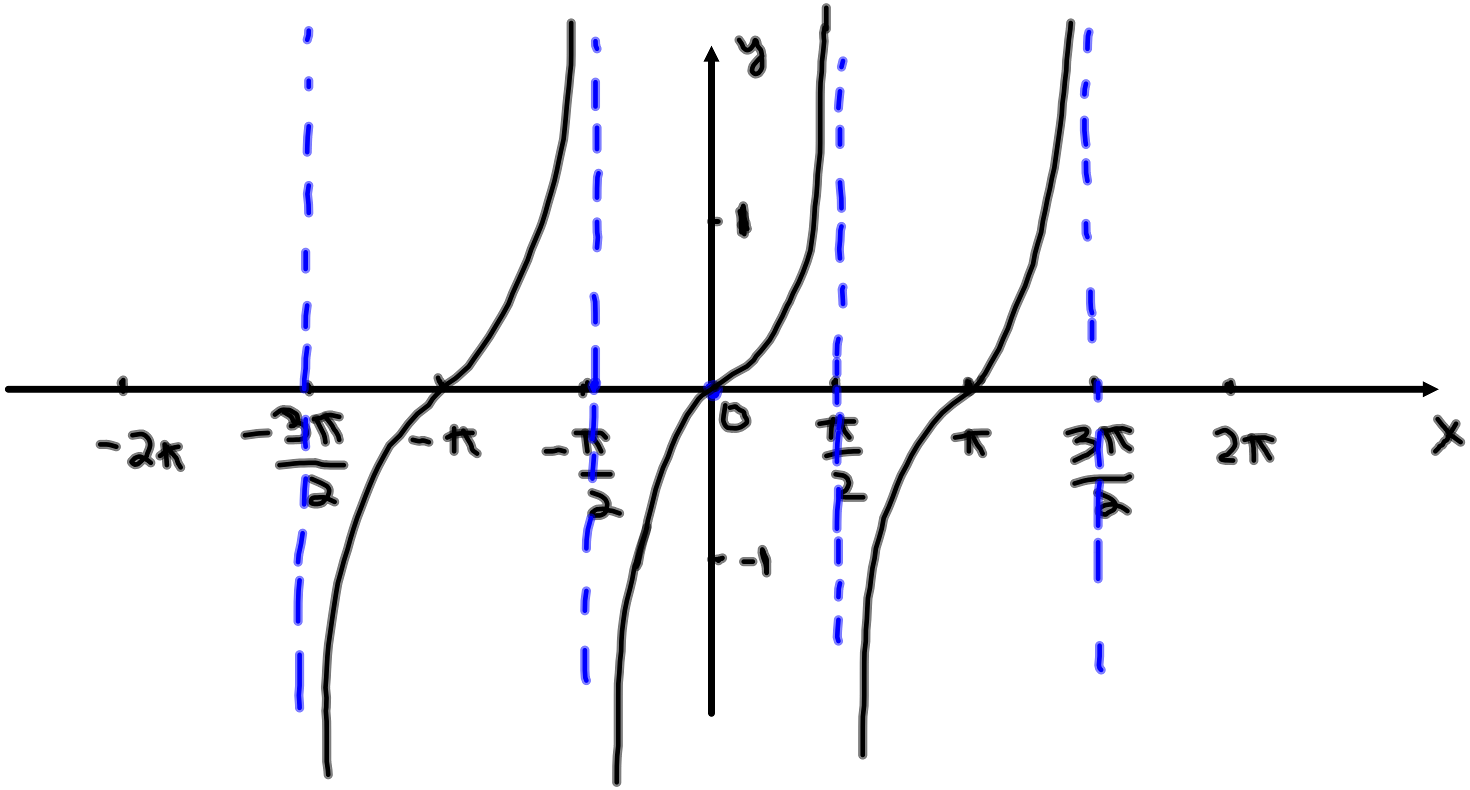
- $y = \sin x$



- $y = \cos x$



- $y = \tan x$

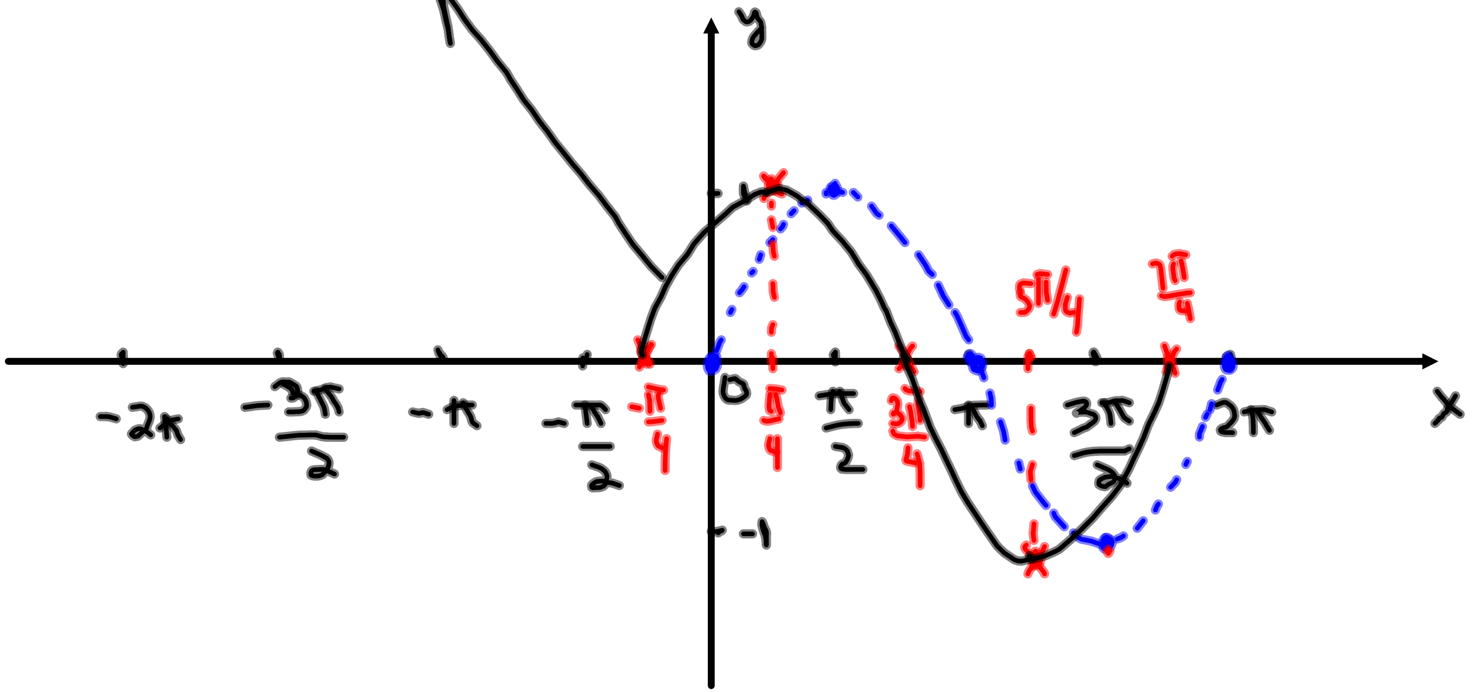


EXAMPLE 7. *Graph the following functions:*

(a) $f(x) = \sin(x + \frac{\pi}{4})$

- - - - - $y = \sin x$

shift left a distance $\frac{\pi}{4}$



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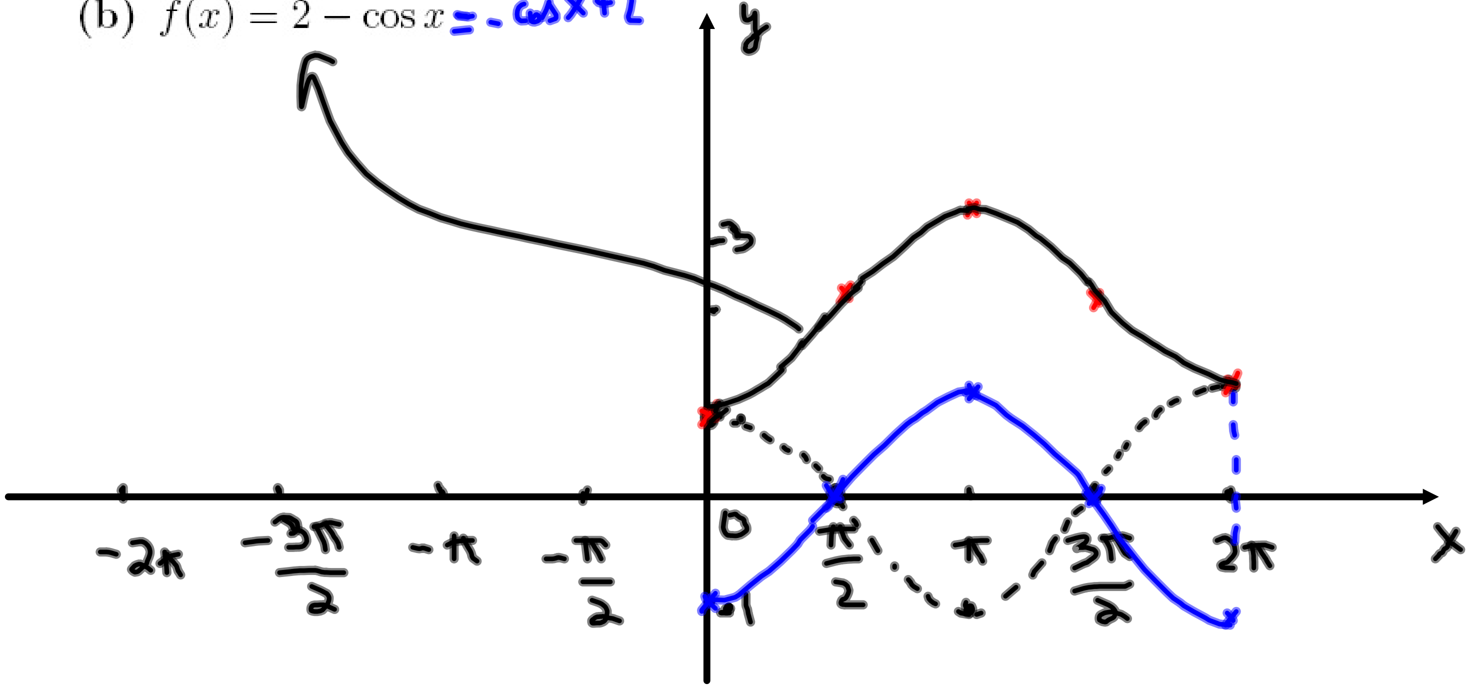
$c > 0 \Rightarrow y = f(x+c)$ shift the graph $y = f(x)$ a distance c left

(b) $f(x) = 2 - \cos x = -\cos x + 2$



$\cos x \rightarrow -\cos x \rightarrow -\cos x + 2$

reflect about the x-axis

Shift up by 2 units