

Approximation and interpolation with polyharmonic and similar kernels

I'll discuss some recent results involving approximation with kernels obtained as fundamental solutions of elliptic differential operators. Such kernels can effectively treat scattered data in a variety of interesting settings. They are a powerful tool for approximation in high dimensions and on esoteric domains like Lie groups, graphs and manifolds. I will discuss some of the settings where precise approximation results are known, and I will detail one of the main problems facing this kind of approximation: the effect of the boundary.

New results will also be discussed concerning the stability of boundary-free interpolation. Under mild assumptions on the geometry of the data, this kind of interpolation can be shown to have a uniformly bounded Lebesgue constant (i.e., the L_∞ operator norm of the projection operator can be bounded independent of the cardinality of the data). However, much stronger results may hold. When the spacing of the data sites is permitted to vary spatially, interpolation is stable in a more refined sense, one that takes into account the local spacing of the data sites. Thus, a certain "penalized" Lebesgue constant (an operator norm on a weighted L_∞ space) is uniformly bounded, too.

Such stability results dovetail nicely with known error estimates – leading to "Lebesgue lemmas" which show that interpolation is near-best approximation. We present one such, showing that the (theoretical) local approximation results obtained recently by DeVore and Ron hold for interpolation as well.