

Math 220 Exam 1
February 16, 2007
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Name _____

There are 5 questions, for a total of 100 points. Point values are written beside each question.

1. Consider the following proposition: For all real numbers x , if $x > 2$, then $x^2 > 4$.
 - (a) [5 points] Write the converse of this proposition.

(b) [5] Write the contrapositive of this proposition.

(c) [5] Write the negation of this proposition.

(d) [5] Which of the above four statements (*the proposition, its converse (a), its contrapositive (b), its negation (c)*) are true? (You need not justify your answer.)

2. (a) [15] Give a truth table for the statement $(P \rightarrow Q) \wedge (P \wedge \neg Q)$.

(b) [5] Is the proposition from part (a) a tautology, contradiction, or neither?

3. Let a and b be integers.

(a) [10] Prove the proposition: If a is divisible by 3 and b is divisible by 3, then ab is divisible by 9.

(b) [10] Is the converse of the proposition given in (a) true? If so, prove it; if not, give a counterexample.

4. [20] Let A and B be sets. Prove that $A \subseteq B$ if and only if $A - B = \emptyset$.

5. For each natural number n , let $A_n = \left[\frac{1}{n}, \frac{n+1}{n} \right]$. In the following, you need not prove that your answers are correct.

(a) [10] Find $A_1 \cup A_2$ and $A_1 \cap A_2$.

(b) [10] Find $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$.

SCRATCH PAPER