Math 220 Quantifiers and Negations

1. For each statement, determine whether it has a universal or existential quantifier, and specify which one. (It may help to rewrite some of the sentences using "for all..." or "there exists...".)

   (a) All triangles are equilateral. universal (False.)

   (b) There exists a triangle that is not equilateral. existential (True)

   (c) The area of a triangle is half its base times its height. (True)
   for all triangles, the area is half the base times the height. universal quantifier

   (d) 0=1. no quantifier (False.)

   (e) There is a smallest positive integer. existential (True: 1 is the smallest positive integer)

   (f) Every odd integer has square an odd integer. universal (True.)

   (g) A differentiable function is continuous.
   Every differentiable function is continuous. universal (True.)

   (h) Some even numbers are multiples of three. existential
   (True: 12 is even, and is a multiple of 3 also 6, 36, ...)

2. State the negation of each of the following statements.

   (a) Some even numbers are multiples of three. (True)
   ~~All~~ No even numbers are multiples of three. (or All even numbers are not multiples of three.) (False.)

   universal quantifier

   (b) All triangles are equilateral. (False.)
   There exists a triangle that is not equilateral.

   (c) There is an odd integer whose square is an even integer. (False.)
   There is ~~not~~ no odd integer whose square is an even integer. Better: No odd integer has square that is an even integer.

   (d) All new cars have something wrong with them.
   Some new cars have nothing wrong with them.

   (e) There are sets that contain infinitely many elements. (True.)
   All sets do not contain infinitely many elements. or: No set contains infinitely many elements.

   (f) A continuous function is differentiable.

   (g) If f is a polynomial function, then f is continuous everywhere. (True)

   (h) For each real number x, there is a real number y such that $y^2 = x$. (False)
   There exists a real number x such that there is ~~not~~ no real number y such that $y^2=x$ → counterexample $x=-1$

   (i) For each real number x, there is a real number y such that $y^3 = x$. (True.)

i.e. the original statement is: All continuous functions are differentiable. (False.)
Negation: There is a continuous function that is not differentiable. (True.)