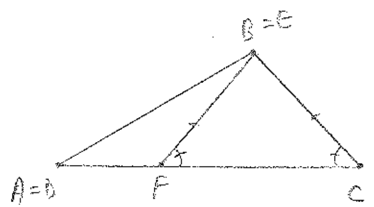
Math 367 In-class Assignment 5

Name Key

1. (a) Draw a picture showing that there is no SSA Congruence Theorem. That is, draw two triangles $\triangle ABC$ and $\triangle DEF$ for which $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\angle BAC \cong \angle EDF$, yet $\triangle ABC \not\cong \triangle DEF$.

Answers will vary.

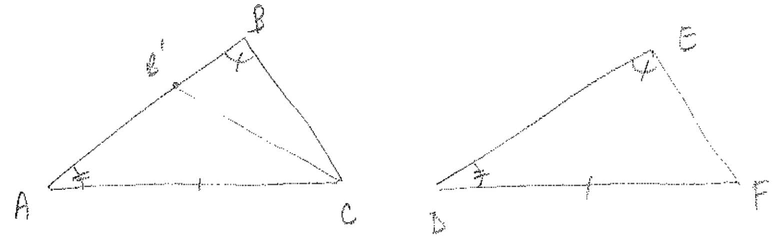
(b) Redraw your picture from part (a), but this time identify some parts of the two triangles in such a way that $A = D$, $B = E$, and $\angle BAC = \angle EDC$. (Think of this as putting one triangle on top of the other.) Use the Isosceles Triangle Theorem to show that $\angle ACB$ and $\angle DFE$ are supplementary. Explain. (Even though there is no SSA Theorem, this shows that there are only these two possibilities.)



By the Isosceles Triangle Theorem, $\angle BFC \cong \angle BCF$. Since $\angle DFE$ and $\angle CFE$ form a linear pair, they are supplementary. Therefore $\angle ACB$, $\angle DFE$ are supplementary.

2. Follow the steps below to prove the AAS Theorem. That is, prove that if $\triangle ABC$ and $\triangle DEF$ are triangles such that $\angle ABC \cong \angle DEF$, $\angle BAC \cong \angle EDF$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

(a) Draw a picture of the two triangles, marking the given congruent angles and sides.



(b) Note that *if* $\overline{AB} \cong \overline{DE}$, then by the SAS Postulate, the two triangles are congruent, so we are finished. Start a proof by contradiction by assuming $\overline{AB} \not\cong \overline{DE}$. By the Point Construction Postulate, there is a point B' on $\overrightarrow{AB}$ such that $\overline{AB'} \cong \overline{DE}$. Put this point B' in your picture. Explain how to use the SAS Postulate to conclude that $\triangle AB'C \cong \triangle DEF$.

Since $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle B'AC \cong \angle EDF$, by the SAS Postulate, $\triangle AB'C \cong \triangle DEF$.

(c) Note that by hypothesis, $\angle ABC \cong \angle DEF$, and now by part (a), $\angle AB'C \cong \angle DEF$. It follows that $\angle ABC \cong \angle AB'C$. Explain how to use the Exterior Angle Theorem to arrive at a contradiction.

$\angle AB'C$ is an exterior angle to $\triangle B'BC$. By the Exterior Angle Theorem, $m(\angle AB'C)$ is greater than the measure of any interior angle of $\triangle B'BC$. However $\angle AB'C \cong \angle B'BC$, a contradiction.