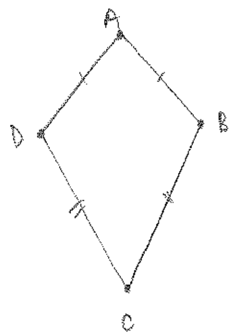
Math 367 In-class Assignment 8

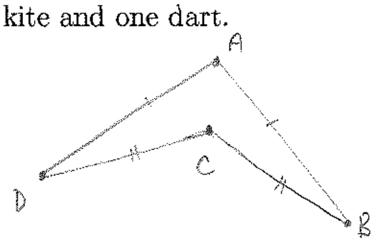
Name Key

Assume the axioms of Euclidean geometry, as in Chapter 5.

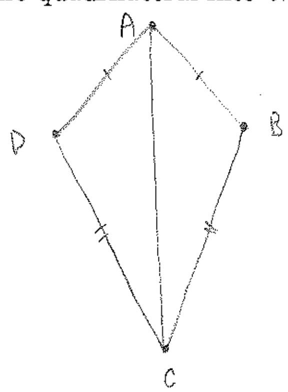
1. Consider a quadrilateral $\square ABCD$ for which $AB = AD$ and $CB = CD$. Such a quadrilateral is called a *kite* (if its interior is convex) or a *dart* (if its interior is not convex).

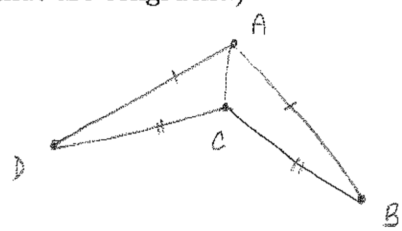
(a) Draw one of each type, that is, draw one kite and one dart.





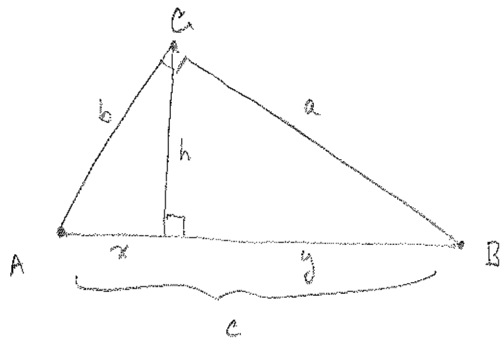
(b) Prove that, for either type, $\angle ABC \cong \angle ADC$. *(Hint: Show that the diagonal $\overline{AC}$ divides the quadrilateral into two triangles that are congruent.)*





Since $AC = AC$, in either case, by the SSS Theorem, $\triangle ABC \cong \triangle ADC$. Therefore $\angle ABC \cong \angle ADC$.

2. Prove Theorem 5.4.4, that is, prove that the length of one leg of a right triangle is the geometric mean of the length of the hypotenuse and the length of the projection of that leg onto the hypotenuse. *(Hint: In the notation of Figure 5.10 in the textbook, this means to prove that $b = \sqrt{cx}$, or equivalently, since these quantities are all positive, this means to prove that $b^2 = cx$. To do this, write cx in a different way that will eliminate c, and use Theorem 5.4.3.)*



By Theorem 5.4.3, $h = \sqrt{xy}$, so $h^2 = xy$.

Now $c = x + y$, so

$$cx = (x+y)x$$
$$= x^2 + xy$$
$$= x^2 + h^2.$$

By the Pythagorean Theorem, $x^2 + h^2 = b^2$, and so

$$cx = b^2.$$