

# Trigonometric Identities

## Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

## Even-Odd Identities

Sine, cosecant, tangent, and cotangent are odd functions; cosine and secant are even functions.

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$

## Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

## Double-Angle Identities

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

## Sum/Difference Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

## **Half-Angle Identities**

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}, \cos \alpha \neq -1$$

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha}, \sin \alpha \neq 0$$

$$\tan \frac{\alpha}{2} = \frac{1 + \sin \alpha}{\cos \alpha}, \cos \alpha \neq -1$$

## **Product/Sum Identities**

$$2 \cos \alpha \cos \beta = \cos (\alpha - \beta) + \cos (\alpha + \beta)$$

$$2 \sin \alpha \sin \beta = \cos (\alpha - \beta) - \cos (\alpha + \beta)$$

$$2 \sin \alpha \cos \beta = \sin (\alpha + \beta) + \sin (\alpha - \beta)$$

$$2 \cos \alpha \sin \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$$

## **Alternate Forms of Product/Sum Identities**

$$\cos x + \cos y = 2 \cos \left( \frac{x + y}{2} \right) \cos \left( \frac{x - y}{2} \right)$$

$$\cos x - \cos y = 2 \sin \left( \frac{x + y}{2} \right) \sin \left( \frac{x - y}{2} \right)$$

$$\sin x + \sin y = 2 \sin \left( \frac{x + y}{2} \right) \cos \left( \frac{x - y}{2} \right)$$

$$\sin x - \sin y = 2 \cos \left( \frac{x + y}{2} \right) \sin \left( \frac{x - y}{2} \right)$$