Math 131 Lecture Notes
Section 7.1 – Modeling with Differential Equations

A **differential equation** is an equation that contains an unknown function and some of its derivatives.

The **order** of a differential equation is the order of the highest derivative that occurs in the equation.

A function f is called a **solution** of a differential equation if the equation is satisfied when $y = f(x)$ and its derivatives are substituted into the equation. Thus f is a solution of the differential equation $y' = xy$ if $f'(x) = xf(x)$ for all values of x in some interval.

To solve a differential equation, we must find all possible solutions of the equation. We have already solved some simple differential equations—those of the form $y' = f(x)$.

In many physical problems, we need to find the particular solution that satisfies an **initial condition,** i.e., a condition of the form $y(t_0) = y_0$.

Geometrically, when an initial condition is imposed, we will choose from the family of solution curves the one that passes through the point (t_0, y_0). This corresponds to measuring the state of a system at time t_0 and using the solution of the initial-value problem to predict the future behavior of the system.

Example: (example 2)
Find a solution of the differential equation $y' = \frac{1}{2}(y^2 - 1)$ that satisfies the initial condition $y(0) = 2$.

<u>Models of Population Growth</u>

One model for population growth is based on the assumption that the population grows at a rate proportional to the size of the population, $\frac{dP}{dt} = kP$, where *t* is the time, *P* is the number of individuals in the population, and *k* is the proportionality constant.

Then $\frac{dP}{dt}$ gives the rate of growth of the population. The exponential function fits the criteria of the rate of growth being proportional to the function itself. So if $P(t) = Ce^{kt}$, then $P'(t) = C(ke^{kt}) = k(Ce^{kt}) = kP(t)$. Since populations have only positive values, we are interested only in the solutions with $C > 0$, and probably only values of *t* greater than the initial time $t = 0$.

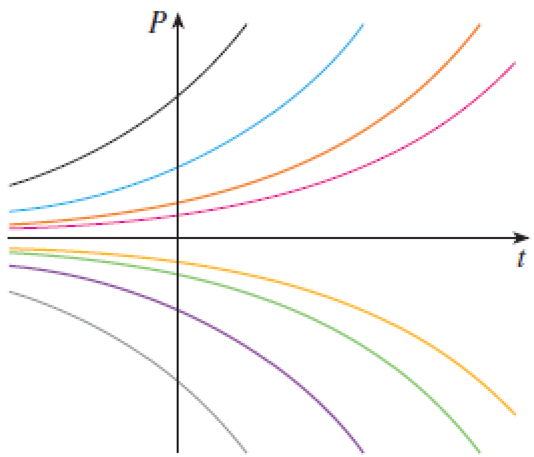

FIGURE 1
The family of solutions of $dP/dt = kP$

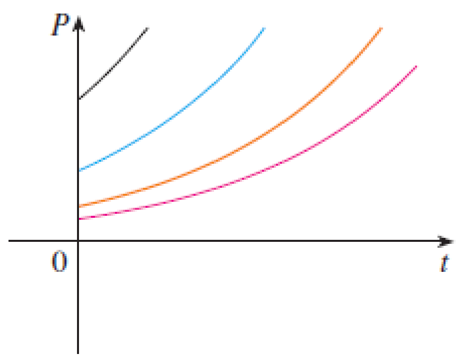

FIGURE 2
The family of solutions $P(t) = Ce^{kt}$

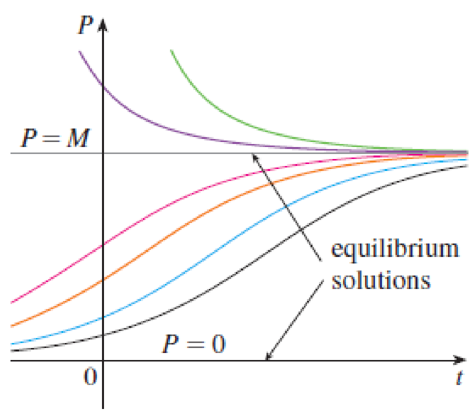

FIGURE 3
Solutions of the logistic equation

Example: #4

a) For what values of k does the function $y = \cos kt$ satisfy the differential equation $4y'' = -25y$?

b) For those values of k, verify that every member of the family of functions $y = A \sin kt + B \cos kt$ is also a solution.