

Math 150 Lecture Notes Exponents and Radicals

Integer Exponents

If a is any real number and n is a positive integer, then the **n th power** of a is $a^n = a \cdot a \cdot \cdots \cdot a$ (n factors of a). The number a is the **base** and n is the **exponent**.

If $a \neq 0$ is any real number and n is a positive integer, then $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$.

Laws of Exponents

$$1. \quad a^m \cdot a^n = a^{m+n}$$

$$2. \quad \frac{a^m}{a^n} = a^{m-n}$$

$$3. \quad (a^m)^n = a^{mn}$$

$$4. \quad (ab)^n = a^n b^n$$

$$5. \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$6. \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$7. \quad \frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$$

Scientific Notation

A positive number x is said to be written in **scientific notation** if it is expressed as follows:

$$x = a \times 10^n \quad \text{where } 1 \leq a < 10, n \text{ is an integer}$$

Radicals

$$\sqrt{a} = b \quad \text{iff} \quad b^2 = a \quad \text{and} \quad b \geq 0$$

If n is any positive integer, then the **principal n th root** of a is defined as:

$$\sqrt[n]{a} = b \quad \text{means} \quad b^n = a$$

Properties of n th Roots

1. $\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$

2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

4. $\sqrt[n]{a^n} = a$ if n is odd

5. $\sqrt[n]{a^n} = |a|$ if n is even

Rational Exponents

For any rational exponent $\frac{m}{n}$ in lowest terms, where m and n are integers and $n > 0$,

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

Rationalizing the Denominator

A useful procedure that involves rewriting an expression without a radical in the denominator (by multiplying both numerator and denominator by an appropriate expression) is called **rationalizing the denominator**. It is sometimes useful, instead, to rationalize the numerator.