Math 150 Lecture Notes
Solving Equations

An **equation** is a statement that two mathematical expressions are equal.

Solving an equation is the process of finding the **solutions** or **roots**, the values for the variable(s) that make the equation true.

Equivalent equations are equations with exactly the same solutions.

Properties of Equality

1. $A = B \Leftrightarrow A + C = B + C$
2. $A = B \Leftrightarrow CA = CB \quad (C \neq 0)$

Linear Equations

A **linear equation in one variable** is an equation equivalent to one of the form

$$ax + b = 0$$

where a and b are real numbers and x is the variable.

Quadratic Equations

A **quadratic equation** is an equation of the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers with $a \neq 0$.

Zero Product Property

$AB = 0$ if and only if $A = 0$ or $B = 0$

The solutions of the equation $x^2 = c$ are $x = \sqrt{c}$ and $x = -\sqrt{c}$.

Completing the Square is a term for a process that puts the quadratic equation in the form above so that the left side of the equation is a perfect square.

The Quadratic Formula

The roots of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **discriminant** of the general quadratic $ax^2 + bx + c = 0$ ($a \neq 0$) is $D = b^2 - 4ac$.

1. If $D > 0$, then the equation has two distinct real solutions.
2. If $D = 0$, then the equation has exactly one real solution.
3. If $D < 0$, then the equation has no real solution.

Other Types of Equations

When we use a process that does not produce equivalent equations to solve an equation (such as one involving a radical where we square both sides of the equation), we may have one or more **extraneous solutions**, which are solutions of the resulting equation but not of the original equation. In these cases, checking the answers in the original equation is not solely for the purpose of finding mistakes in our work but a necessary part of the process to eliminate extraneous solutions.

An equation of **quadratic type** is one of the form $aW^2 + bW + c = 0$, where W is an algebraic expression.

Example 1: $\frac{4}{x-1} + \frac{2}{x+1} = \frac{35}{x^2-1}$

Example 2: Solve for n: $S = \frac{n(n+1)}{2}$

Example 3: $-2x^2 + 6x + 3 = 0$

Example 4: $\sqrt{\sqrt{x-5} + x} = 5$

Example 5: $\sqrt{x} - 3\sqrt[4]{x} - 4 = 0$