Math 150 Lecture Notes
The Cartesian Coordinate System

The Coordinate Plane

The **Cartesian Coordinate System** is formed by two perpendicular number lines that intersect at 0 on each line. The horizontal axis is the ***x*-axis**; the vertical axis is the ***y*-axis**. The point of intersection is the **origin**. The two axes divide the plan into four **quadrants**. Any point P in the coordinate plane can be described by a unique **ordered pair** of numbers (a, b), for which a is the ***x*-coordinate** or **abscissa** and b is the ***y*-coordinate** or **ordinate**.

Distance and Midpoint

The **distance between two points**, $A(x_1, y_1)$ and $B(x_2, y_2)$ is $d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

The **midpoint** of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Graphs of Equations in Two Variables

An **equation in two variables** expresses a relationship between two quantities. A point (x, y) **satisfies** the equation if it makes the equation true when the values for x and y are substituted into the equation.

The **graph** of an equation in x and y is the set of all points (x, y) in the coordinate plane that satisfy the equation.

Intercepts

The ***x*-intercept** is the point where the graph of an equation intersects the x-axis.

The ***y*-intercept** is the point where the graph of an equation intersects the y-axis.

Circles

A **circle** is the set of all points $P(x, y)$ in a plane that are the same distance r from a given point $C(h, k)$, which is the **center** of the circle. The distance r is the **radius** of the circle.

The **standard form** for the **equation of the circle** with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

If the center of the circle is the origin (0, 0), then the equation is $x^2 + y^2 = r^2$.

Symmetry

When a graph is s**ymmetric with respect to the *x*-axis**, it is a mirror image of itself across the x-axis; when y is replaced with $-y$ in the equation, the result (after simplification) is the same equation. For every point (x, y) on the graph, the point $(x, -y)$ is on the graph.

When a graph is **symmetric with respect to the *y*-axis**, it is a mirror image of itself across the y-axis; when x is replaced by $-x$ in the equation, the result (after simplification) is the same equation. For every point (x, y) on the graph, the point $(-x, y)$ is on the graph.

When a graph is **symmetric with respect to the origin**, it gives the same graph when it is rotated 180° about the origin or when it is reflected across the x-axis, then the y-axis (or the y-axis, then the x-axis). When x is replaced by $-x$ <u>and</u> y is replaced by $-y$, the resulting equation is the same (after simplification). For every point (x, y) on the graph, the point $(-x, -y)$ is on the graph.