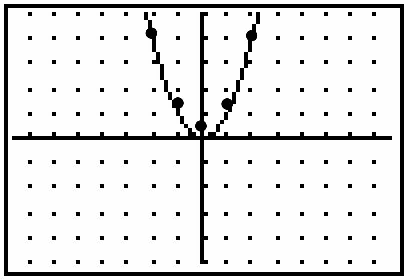
Math 150 Lecture Notes
Transformations of Functions



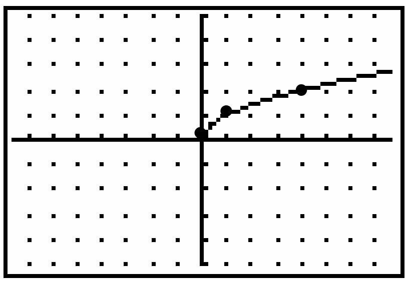
Quadratic Function

$f(x) = x^2$

Anchor Points: (-1, 1, (0, 0), (1, 1), (-2, 4), (2, 4)

D = $\{x|\ x \in \mathrm{R}\}$ or $(-\infty, \infty)$

R = $\{x|\ x \in \mathrm{R},\ x \geq 0\}$ or $[0, \infty)$



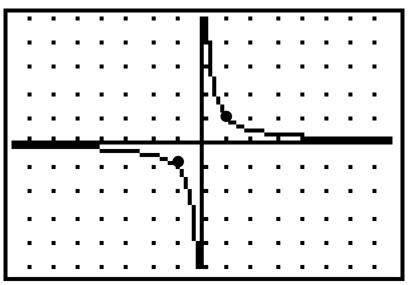
Square Root Function

$g(x) = \sqrt{x}$

Anchor Points: (0, 0), (1, 1), (4, 2)

D = $\{x|\ x \in \mathrm{R},\ x \geq 0\}$ or $[0, \infty)$

R = $\{x|\ x \in \mathrm{R},\ x \geq 0\}$ or $[0, \infty)$



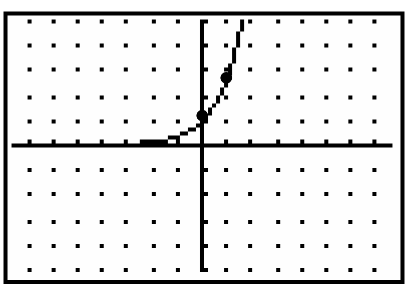
Rational Function

$h(x) = \dfrac{1}{x}$

Anchor Points: (-1, 1), (1, 1)

D = $\{x|\ x \in \mathrm{R},\ x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$

R = $\{x|\ x \in \mathrm{R},\ x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$



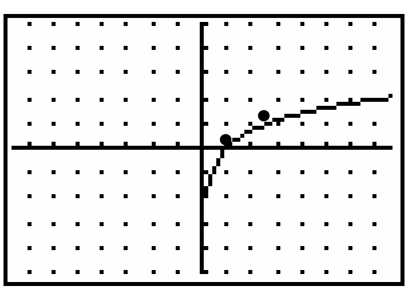
Exponential Function

$F(x) = e^x$

Anchor Points: (0, 1), (1, e)

D = $\{x|\ x \in \mathrm{R}\}$ or $(-\infty, \infty)$

R = $\{x|\ x \in \mathrm{R},\ x > 0\}$ or $(0, \infty)$



Logarithmic Function

$G(x) = \ln x$

Anchor Points: (1, 0), (e, 1)

D = $\{x|\ x \in \mathrm{R},\ x > 0\}$ or $(0, \infty)$

R = $\{x|\ x \in \mathrm{R}\}$ or $(-\infty, \infty)$

Summarize in a sentence.

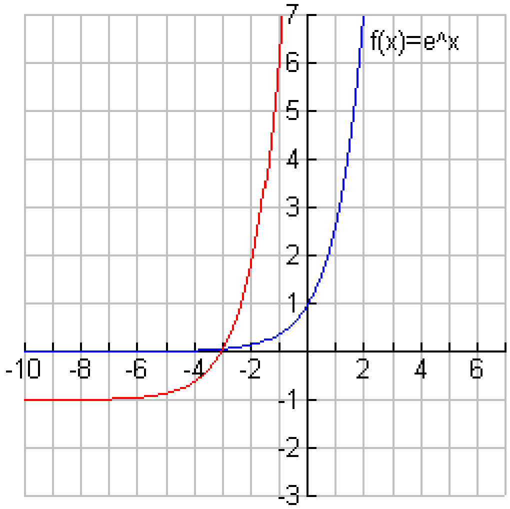
Equation	Effect
1. $y=f(x+2)$	*The graph of* $f(x+2)$ *is the graph of* $f(x)$ *translated left* 2.
2. $y=f(x-2)$	*The graph of* $f(x-2)$*is the graph of* $f(x)$ *translated right* 2.
3. $y=f(x)+2$	*The graph of* $f(x)+2$ *is the graph of* $f(x)$*translated up* 2.
4. $y=f(x)-2$	*The graph of* $f(x)-2$ *is the graph of* $f(x)$ *translated down* 2.
5. $y=f(2x)$	*The graph of* $f(2x)$ *is the graph of* $f(x)$ *horizontally compressed or shrunk by a factor of* 2.
6. $y=f\left(\frac{1}{2}x\right)$	*The graph of* $f\left(\frac{1}{2}x\right)$ *is the graph of* $f(x)$ *horizontally stretched by a factor of* $\frac{1}{2}$.
7. $y=2f(x)$	*The graph of* $2f(x)$ *is the graph of* $f(x)$ *vertically stretched by a factor of* 2.
8. $y=\frac{1}{2}f(x)$	*The graph of* $\frac{1}{2}f(x)$ *is the graph of* $f(x)$ *vertically compressed or shrunk by a factor of* $\frac{1}{2}$.
9. $y=f(-x)$	*The graph of* $f(-x)$ *is the graph of* $f(x)$ *reflected across the y-axis*.
10. $y=-f(x)$	*The graph of* $-f(x)$ *is the graph of* $f(x)$ *reflected across the x-axis*.
11. $y=\lvert f(x)\rvert$	*The graph of* $\lvert f(x)\rvert$ *is the graph of* $f(x)$ *where* $f(x)\geq 0$ *and the graph of* $-f(x)$ *where* $f(x)<0$. *In other words, the points with positive y-values remain the same and the points with negative y-values are reflected across the x-axis.*
12. $y=f(\lvert x\rvert)$	*For the graph of* $f(\lvert x\rvert)$*when x values are greater than or equal to zero, the graph remains the same, is reflected across the y-axis, and replaces the values where* $x<0$.

f is an **even function** if *f*(-*x*) = *f*(*x*) for all *x* in the domain of *f*.

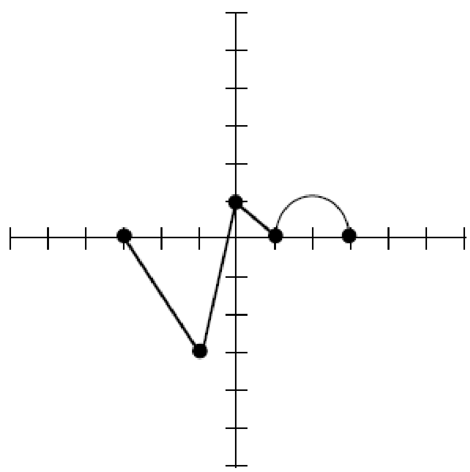
f is an **odd function** if *f*(-*x*) = -*f*(*x*) for all *x* in the domain of *f*.

Example 1: Describe how the graph of $y = 2f(x - 2) + 2$ can be obtained from the graph of f.

Example 2: The graphs of f and g are given. Fund the function rule for g.



Example 3: The graph of f is given. Sketch the graph of $y = -f(x - 1) + 3$.



Example 4: Determine whether the function f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

$$f(x) = 3x^4 - 2x^2$$

Example 5: Determine whether the function f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

$$f(x) = 3x^3 + 2x^2 + 1$$