Math 150 Lecture Notes
Maxima and Minima

A **maximum** of **minimum** value of a function is the largest or smallest value of the function an interval.

A **quadratic function** is a function f of the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$. The graph of any quadratic function is a **parabola**.

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **standard form**

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k); the parabola opens upward if $a > 0$ or downward if $a < 0$.

Let f be a quadratic function with standard form $f(x) = a(x - h)^2 + k$. The maximum or minimum value of f occurs at $x = h$.

If $a > 0$, then the **minimum value** of f is $f(h) = k$.
If $a < 0$, then the **maximum value** of f is $f(h) = k$.

Example 1: Express the given quadratic function in standard form. Find the vertex and x- and y-intercepts. Sketch the graph.

$$f(x) = -3x^2 + 6x - 2$$

Example 2: Find the vertex of the general quadratic function $f(x) = ax^2 + bx + c$.

The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at $x = -\frac{b}{2a}$.

Example 3: Find the local maximum and minimum values of the function and the value of x at which each occurs. State the answer correct to two decimal places.

$$f(x) = x\sqrt{x - x^2}$$

Example 4: Find the local maximum and minimum values of the function and the value of x at which each occurs. State the answer correct to two decimal places.

$$g(x) = \frac{1}{x^2 + x + 1}$$