

Math 150 Lecture Notes Composition of Functions

Let f and g be functions with domains A and B . Then the functions $f + g$, $f - g$, fg , and f/g are defined as follows:

$$(f + g)(x) = f(x) + g(x)$$

Domain $A \cap B$

$$(f - g)(x) = f(x) - g(x)$$

Domain $A \cap B$

$$(fg)(x) = f(x)g(x)$$

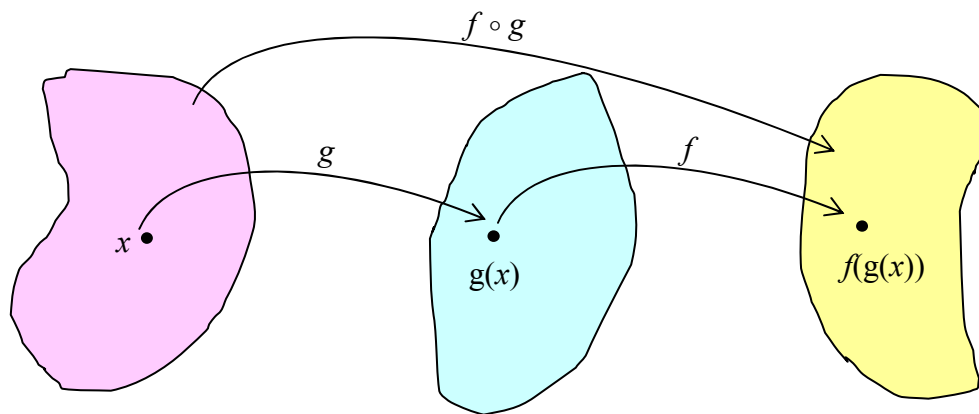
Domain $A \cap B$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Domain $\{x \in A \cap B \mid g(x) \neq 0\}$

Given two functions f and g , the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by $(f \circ g)(x) = f(g(x))$.

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f .



Example 1: Find $f + g$, $f - g$, fg , and f/g and their domains.

$$f(x) = \sqrt{x^2 - 4}$$

$$g(x) = \frac{2}{x+2}$$

Example 2: For $f(x) = 3x - 5$ and $g(x) = 1 - x^2$, evaluate the following.

$$g(g(3)) \quad (g \circ f)(-3) \quad (f \circ g)(-3) \quad (g \circ f)(x)$$

Example 3: Find the functions $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$, and $f \circ g \circ h$, and their domains.

$$f(x) = x - 5 \quad g(x) = \sqrt{x} \quad h(x) = \frac{1}{x+3}$$

Example 4: Express the function $F(x) = \frac{2}{(3 + \sqrt{x})^2}$ in the form $f \circ g \circ h$.