

Math 150 Lecture Notes Inverses and 1-1 Functions

The **inverse** of a function is a rule that acts on the output of the function and produces the corresponding input. It “undoes” or reverses what the function did. Not all inverses of functions are themselves functions.

A function with domain A is called a **one-to-one function** if

$$f(x_1) \neq f(x_2) \quad \text{whenever } x_1 \neq x_2$$

Horizontal Line Test

A function is one-to-one iff no horizontal line intersects its graph more than once.

If f is a one-to-one function with domain A and range B , then its **inverse function** f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x \Leftrightarrow f(x) = y$.

Inverse Function Property

Let f be a one-to-one function with domain A and range B . The inverse function f^{-1} satisfies the following properties:

$$\begin{aligned} f^{-1}(f(x)) &= x && \text{for every } x \text{ in } A \\ f(f^{-1}(x)) &= x && \text{for every } x \text{ in } B \end{aligned}$$

Conversely, any function f^{-1} satisfying these equations is the inverse of f .

The **graph of f^{-1}** can be obtained by reflecting the graph of f across the line $y = x$.

Example 1: Determine whether the function is one-to-one: $f(x) = x^4 + 5$, $0 \leq x \leq 2$

Example 2: Assume f is a one-to-one function. If $f^{-1}(4) = 2$, find $f(2)$.

Example 3: If $f(x) = x + 4x$ with $x \geq -2$, find $f^{-1}(5)$.

Example 4: Find the inverse function of $f(x) = \sqrt{2x-1}$.

Example 5: Draw the graph of f and use it to determine whether the function is one-to-one.

$$f(x) = x|x|$$

Example 6: The given function is not one-to-one. Restrict its domain so that the resulting function **is** one-to-one. Find the inverse of the function with the restricted domain.

$$F(x) = 2 - x^2$$