A polynomial function of degree $n$ is a function of the form
$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$
Where $n$ is a nonnegative integer and $a_n \neq 0$.

The numbers $a_0, a_1, a_2, \ldots, a_n$ are called the **coefficients** of the polynomial.

The number $a_0$ is the **constant coefficient** or **constant term**.

The number $a_n$, the coefficient of the highest power, is the **leading coefficient**, and the term $a_n x^n$ is the **leading term**.

**Graphs of polynomial functions** are smooth curves with no breaks or corners.

The **end behavior** of a polynomial is a description of what happens as $x$ becomes large in the positive or negative direction.

**Notation:**
- $x \to \infty$ means “$x$ becomes large in the positive direction”
- $x \to -\infty$ means “$x$ becomes large in the negative direction”

For any polynomial, the end behavior is determined by the term that contains the highest power of $x$. 
If $P$ is a polynomial function, then $c$ is called a zero of $P$ if $P(c) = 0$.
In other words, the zeros of $P$ are the solutions or roots of the polynomial equation $P(x) = 0$.

If $P$ is a polynomial and $c$ is a real number, then the following are equivalent.
1. $c$ is a zero of $P$.
2. $x = c$ is a solution of the equation $P(x) = 0$.
3. $x - c$ is a factor of $P(x)$.
4. $x = c$ is an $x$-intercept of the graph of $P$.

**Intermediate Value Theorem for Polynomials**
If $P$ is a polynomial function and $P(a)$ and $P(b)$ have opposite signs, then there exists at least one value $c$ between $a$ and $b$ for which $P(c) = 0$.

**Graphing Polynomials Functions**
1. Find all the real zeros or $x$-intercepts. Break it down into linear factors by factoring methods and/or quadratic formula.
2. Plot the $x$-intercepts and determine the shape near the intercepts according to the multiplicity of the factor.
3. Determine the end behavior of the polynomial.
4. Make a table of values including test points to determine where the graph is above or below the $x$-axis and $y$-intercept.
5. Plot the test points and $y$-intercept and sketch a smooth curve passing through the points and having the required end behavior.

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is a polynomials of degree $n$, then the graph of $P$ has at most $n - 1$ local extrema.

**Example 1:** Sketch the graph of the function by transforming the graph of the parent function. Indicate all $x$- and $y$-intercepts on the graph.

$$f(x) = -\frac{1}{2} (x - 2)^5 + 16$$
Example 2: Sketch the graph of the polynomial function: $P(x) = (x - 1)^2 (x + 2)^3$

Example 3: Sketch the graph of the polynomial function: $P(x) = \frac{1}{4} (2x^4 + 3x^3 - 16x - 24)^2$

Example 4: Determine the end behavior of $P$: $-x^5 + 2x^2 + x$

Example 5: Graph the polynomial and determine how many local maxima and minima it has. $y = 1.2x^3 + 3.75x^4 - 7x^3 - 15x^2 + 18x$