Math 150 Lecture Notes
Division of Polynomials

If $P(x)$ and $D(x)$ are polynomials, with $D(x) \neq 0$, then there exist unique polynomials $Q(x)$ and $R(x)$, where $R(x)$ is either 0 or of degree less then the degree of $D(x)$, such that

$$P(x) = D(x) \cdot Q(x) + R(x)$$

The polynomials $P(x)$ and $D(x)$ are called the **dividend** and **divisor**, respectively, $Q(x)$ is the **quotient**, and $R(x)$ is the **remainder**.

The long division process for polynomials mirrors the process for long division of integers.

Synthetic division is a quick method of dividing polynomials (writing only the essential parts) that can be used when the divisor is of the form $x - c$.

Remainder Theorem
If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

Factor Theorem
c is a zero of P iff $x - c$ is a factor of $P(x)$.

Example 1: Divide $P(x)$ by $D(x)$. $P(x) = x^5 + x^4 - 2x^3 + x + 1$ $D(x) = x^2 + x - 1$

Example 2: Divide $P(x)$ by $D(x)$. $P(x) = 6x^4 + 10x^3 + 5x^2 + x + 1$ $D(x) = 3x + 2$