

## Math 150 Lecture Notes

### Rational Functions

A **rational function** is a function of the form  $r(x) = \frac{P(x)}{R(x)}$  where  $P$  and  $Q$  are polynomials.

The **domain of a rational function** consists of all real number  $x$  except those for which the denominator is zero.

The line  $x = a$  is a **vertical asymptote** of the function  $f$  if  $f(x)$  approaches  $\pm \infty$  as  $x$  approaches  $a$  from the right or left.

The line  $y = b$  is a **horizontal asymptote** of the function  $f$  if  $f(x)$  approaches  $b$  as  $x$  approaches  $\pm \infty$ .

#### Asymptotes of Rational Functions

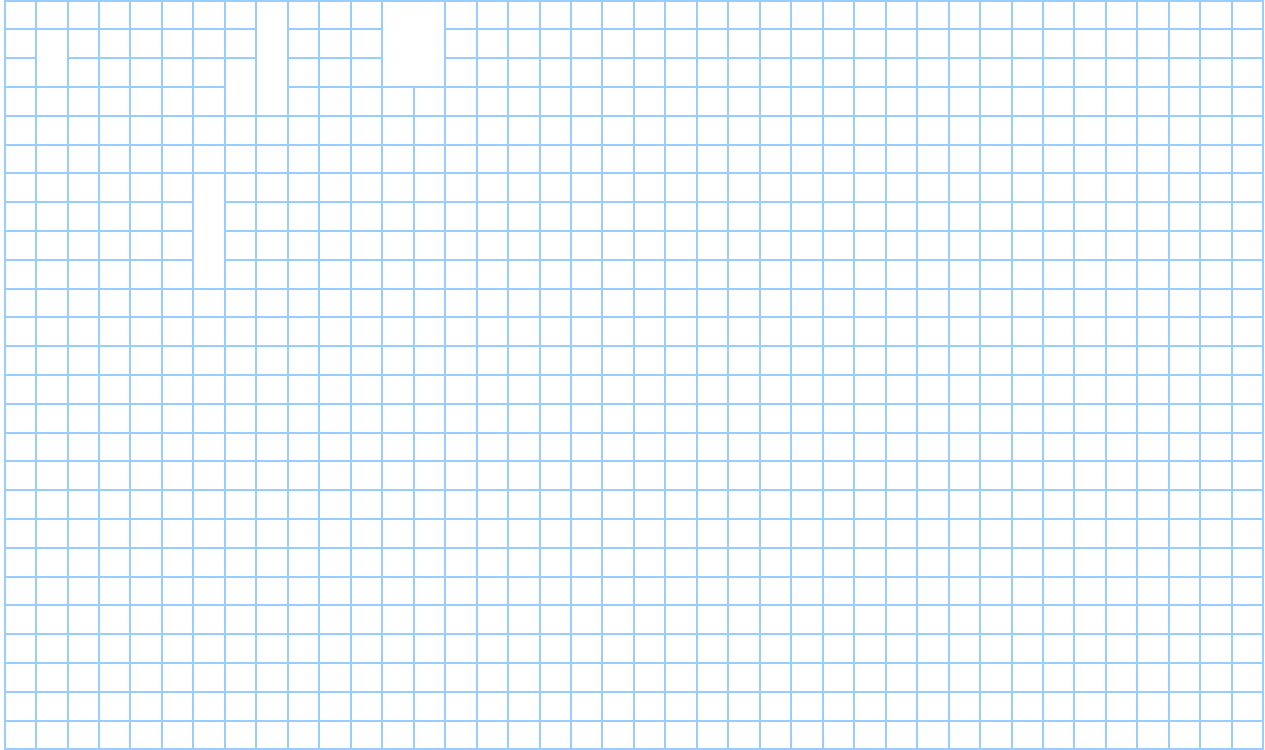
Let  $r$  be the rational function  $r(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0}$

1. The vertical asymptotes of  $r$  are the lines  $x = a$  where  $a$  is a zero of the denominator.
2. a. If  $n < m$ , then  $r$  has horizontal asymptote  $y = 0$ .  
 b. If  $n = m$ , then  $r$  has horizontal asymptote  $y = \frac{a_n}{b_m}$ .  
 c. If  $n > m$ , then  $r$  has no horizontal asymptote.

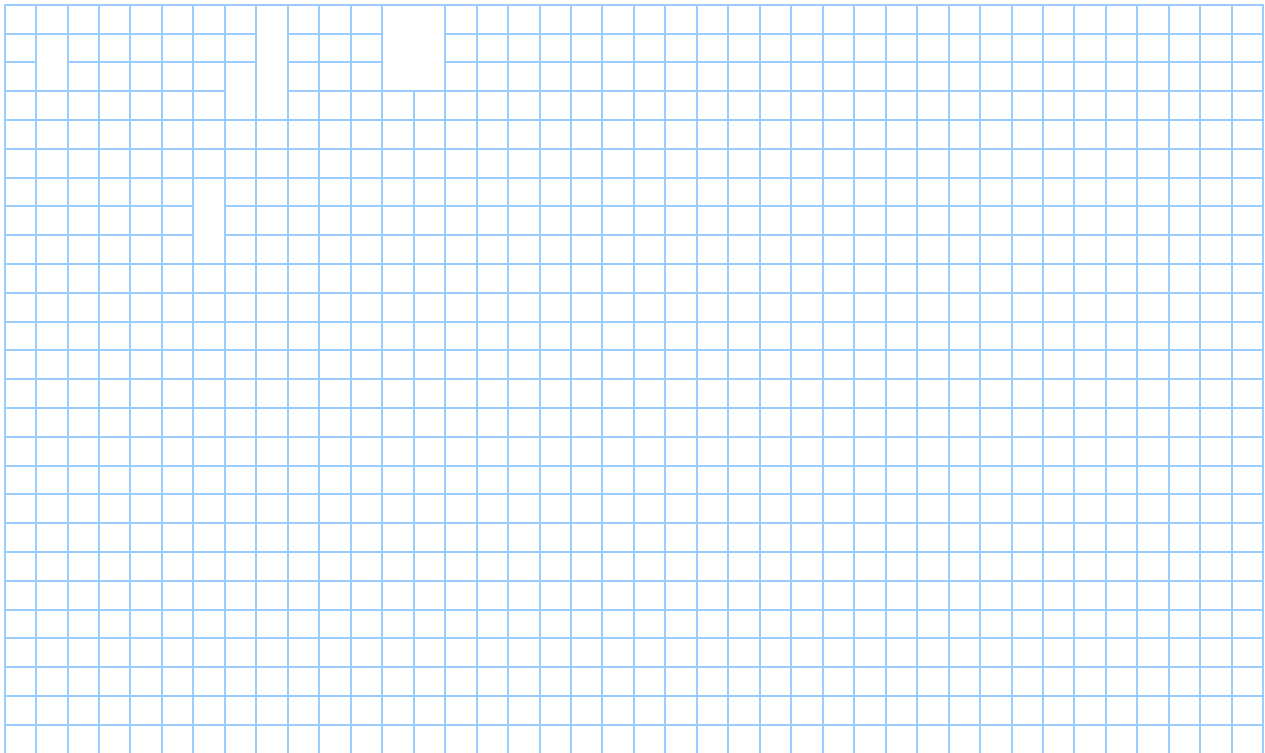
#### To Sketch a Graph of a Rational Function

1. Factor the numerator and denominator. Note that a factor common to numerator and denominator indicates where there is a “hole” in the graph.
2. Find  $x$ - and  $y$ -intercepts and graph them.
3. Find any vertical asymptotes and graph them with dotted lines.
4. Find the horizontal/oblique asymptote and graph it with a dotted line.
5. Make a table of values including test points to determine behavior near asymptotes and plot additional points as needed to determine the rest of the graph.

Example 1: Sketch the graph:  $r(x) = \frac{x^3 - x^2}{x^3 - 3x - 2}$



Example 2: Sketch the graph:  $r(x) = \frac{x^3 + 4}{2x^2 + x - 1}$



Example 3: Sketch the graph:  $r(x) = \frac{-x^4 + 2x^3 - 2x}{(x-1)^2}$

