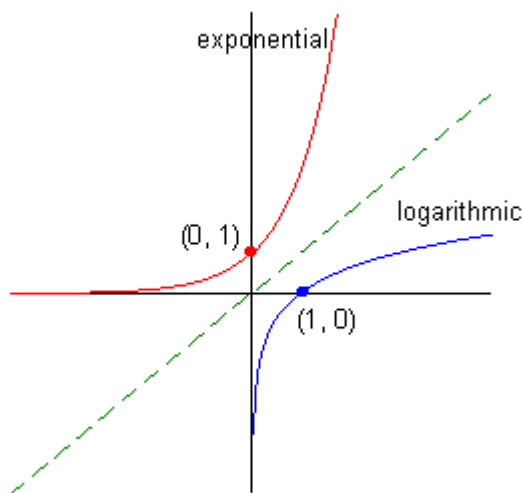


Math 150 Lecture Notes Logarithmic Functions

Every exponential function is a 1-1 function and therefore has an inverse function, the logarithmic function, $f(x) = \log_a x$ ($a > 0, a \neq 1$) with domain $(0, \infty)$ and range $(-\infty, \infty)$. The line $x = 0$ (the y -axis) is a vertical asymptote of f .



The **logarithmic function** with base a , where $a > 0$ and $a \neq 1$, denoted by \log_a , is defined by

$$\log_a x = y \Leftrightarrow a^y = x$$

So, $\log_a x$ is the *exponent* to which the base a must be raised to give x .

Properties of Logarithms

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$
4. $a^{\log_a x} = x$

The logarithm with base 10 is called the **common logarithm** and is denoted by omitting the base:

$$\log x = \log_{10} x$$

The logarithm with base e is called the **natural logarithm** and is denoted by **ln**:

$$\ln x = \log_e x$$

Properties of Natural Logarithms

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^x = x$
4. $e^{\ln x} = x$

Example 1: Write the exponential form of $\log_8 4 = \frac{2}{3}$.

Example 2: Express the equation in logarithmic form: $4^{\frac{3}{2}} = 0.125$

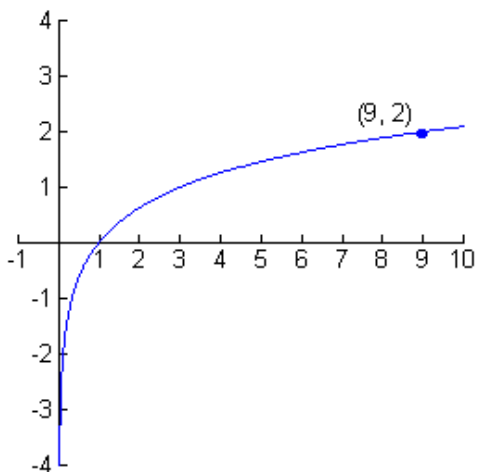
Example 3: Evaluate the following:

$$\log_5 54 \quad \log_2 64 \quad \log_4 64 \quad \log_5 0.2 \quad \log_4 8 \quad \log_4 \frac{1}{2}$$

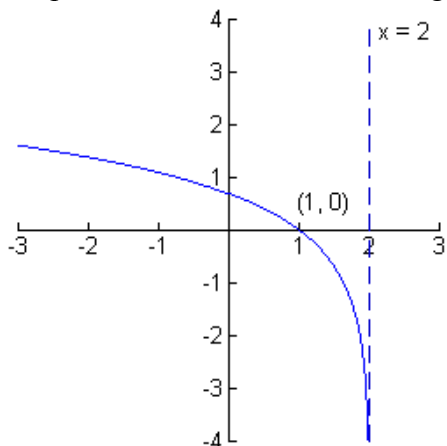
Example 4: Use the definition of the logarithmic function to find x :

$$\log_3 x = 3 \quad \log_3 243 = x \quad \log_x 1000 = 3 \quad \log_x 6 = \frac{1}{2} \quad \log_x 3 = \frac{1}{3}$$

Example 5: Find the function of the form $y = \log_a x$ whose graph is given.



Example 6: Which of the following is the function graphed below?



$$f(x) = -\ln x$$

$$f(x) = \ln(x - 2)$$

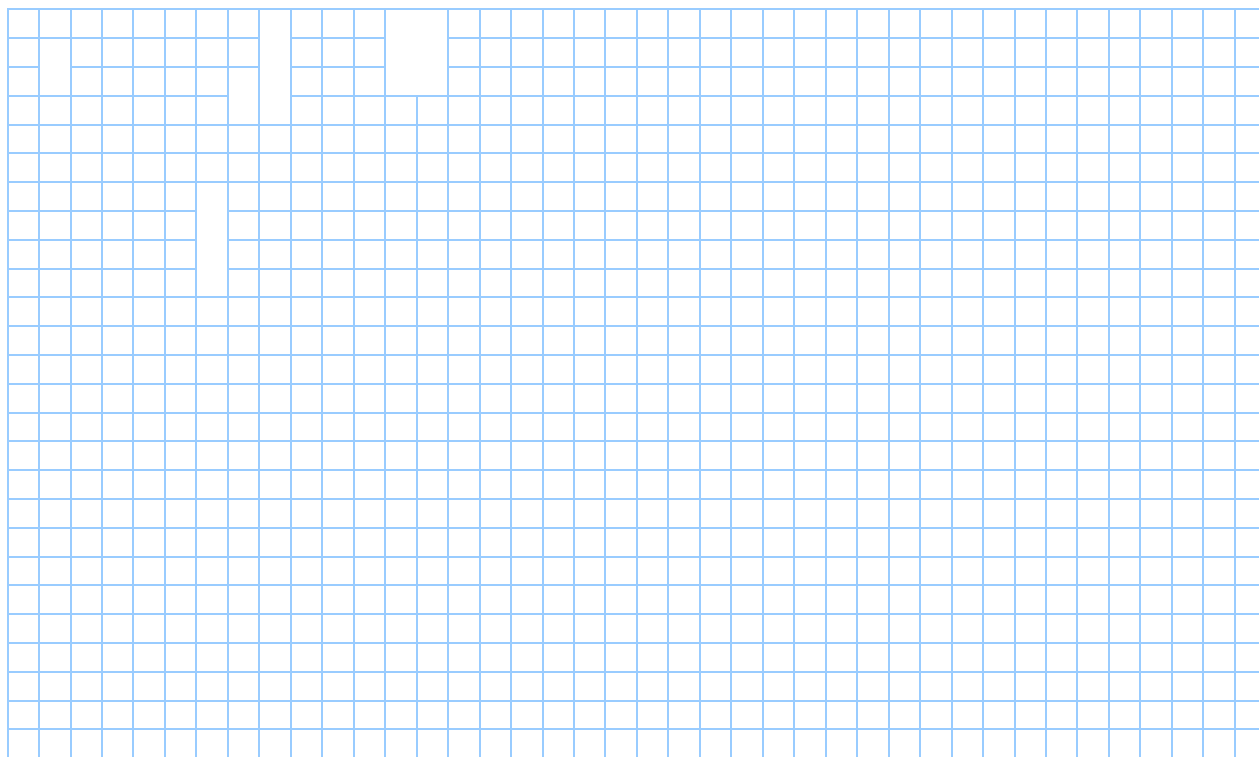
$$f(x) = 2 + \ln x$$

$$f(x) = \ln(-x)$$

$$f(x) = \ln(2 - x)$$

$$f(x) = -\ln(-x)$$

Example 7: Graph the function using transformations of the parent function: $f(x) = \log_3(x - 1) - 2$
 State the domain, range, and asymptote.



Example 8: Find the domain of the function: $f(x) = \log_5 (8 - 2x)$

Example 9: A certain strain of bacteria divides every three hours. If a colony is started with 50 bacteria, then the time t (in hours) required for the colony to grow to N bacteria is given by

$$t = 3 \frac{\log \frac{N}{50}}{\log 2}$$

Find the time required for the colony to grow to a million bacteria.