

Math 150 Lecture Notes

Applications of Exponentials and Logarithms

A population that experiences **exponential growth** increases according to the model

$$n(t) = n_0 e^{rt}$$

where $n(t)$ = population at time t
 n_0 = initial size of the population
 r = relative rate of growth expressed as a proportion of the population
 t = time

If m_0 is the initial mass of a **radioactive substance with half-life** h , then the mass remaining at time

t is modeled by the function $m(t) = m_0 e^{-rt}$ where $r = \frac{\ln 2}{h}$

According to **Newton's Law of Cooling**, if D_0 is the initial temperature difference between an object and its surrounding, and if its surroundings have temperature T_s , then the temperature of the object at time t is modeled by the function $T(t) = T_s + D_0 e^{-kt}$ where k is a positive constant that depends on the type of object.

Chemists measure the acidity of a solution on the **pH Scale**: $\text{pH} = -\log[\text{H}^+]$ where $[\text{H}^+]$ is the concentration of hydrogen ions measured in moles per liter (M).

According to the **Richter Scale**, the magnitude M of an earthquake is $M = \log \frac{I}{S}$, where I is the intensity of the earthquake measured by the amplitude of a seismograph reading taken 100 km from the epicenter of the earthquake and S is the intensity of an earthquake whose amplitude is 1 micron (10^{-4} cm).

The **intensity level** B , measured in decibels (dB), is $B = 10 \log \frac{I}{I_0}$.

Example 1: A culture contains 1500 bacteria initially and doubles every 30 minutes.

- (a) Find a function that models the number of bacteria $n(t)$ after t minutes.
- (b) Find the number of bacteria after 2 hours.
- (c) After how many minutes will the culture contain 4000 bacteria?

Example 2: After 3 days a sample of radon-222 has decayed to 58% of its original amount.

- (a) What is the half-life of radon-222?
- (b) How long will it take the sample to decay to 20% of its original amount?

Example 3: A kettle full of water is brought to a boil in a room with temperature 20°C . After 15 minutes the temperature of the water has decreased from 100°C to 75°C . Find the temperature after another 10 minutes. Illustrate by graphing the temperature function.

Example 4: The Northridge, California earthquake of 1994 had a magnitude of 6.8 on the Richter scale. A year later, a 7.2-magnitude earthquake struck Kobe, Japan. How many times more intense was the Kobe earthquake than the Northridge earthquake?