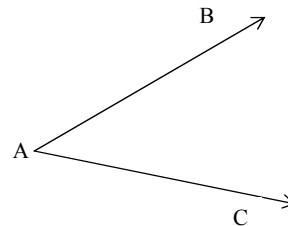


Math 150 Lecture Notes Angle Measure

An **angle** consists of two rays with a common vertex.

For $\angle BAC$ above, we can interpret the angle as a rotation of ray AC onto ray AB, where ray AC is the **initial side** and ray AB is the **terminal side** of the angle. A counterclockwise rotation is **positive**, and a clockwise rotation is **negative**.

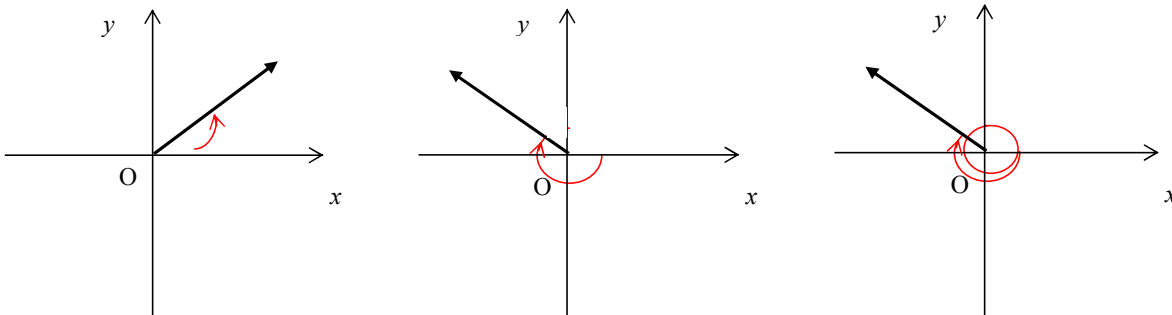


The **measure** of an angle is the amount of rotation about the vertex required to move ray AC onto ray AB. One unit of measurement for angles is the **degree**, which is the amount of rotation that is $\frac{1}{360}$ of a complete revolution. Another measure is the **radian**, which indicates the length of the arc of a circle of radius 1 with center at the vertex of the angle.

Since a complete revolution is measured in 360° or 2π radians, we have the following relationships:

$$180^\circ = \pi \text{ rad} \qquad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ \qquad 1^\circ = \frac{\pi}{180} \text{ rad}$$

An angle is in **standard position** if it is drawn in the xy -plan with its vertex at the origin and its initial side on the positive x -axis.



Two angles in standard position are **coterminal** if their sides coincide.

In a circle of radius r , the **length s of an arc** that subtends a central angles of θ radians is $s = r\theta$.

In a circle of radius r , the area A of a sector with a central angle of θ radians is $A = \frac{1}{2}r^2\theta$.

Linear speed is the rate at which the distance traveled, distance traveled divided by time elapsed.

$$v = \frac{s}{t} = \frac{r\theta}{t} \qquad v = r\omega$$

Angular speed is the rate at which the central angle is changing, angle change divided by time elapsed.

$$\omega = \frac{\theta}{t}$$

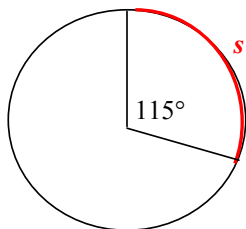
Example 1: Find the radian measure of the angle of 350° .

Example 2: Find the degree measure of the angle with radian measure $\frac{3\pi}{20}$.

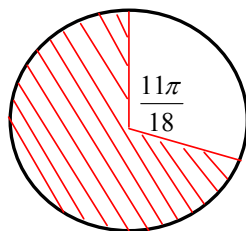
Example 3: Find two positive angles and two negative angles that are coterminal with an angle of measure -50° .

Example 4: Find an angle between 0° and 360° that is coterminal with an angle of -110° .

Example 5: Find the length of arc s in the figure.



Example 6: Find the area of the sector shown in the figure.



Example 7: How many revolutions will a bicycle wheel of diameter 28 in. make as the bicycle travels a distance of 1 mile?