Math 150 Lecture Notes
Trigonometric Functions of Angles

Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side.
If $r = \sqrt{x^2 + y^2}$ is the distance from the origin to the point $P(x, y)$, then

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \ (x \neq 0)$$

$$\csc\theta = \frac{r}{y} \ (y \neq 0) \qquad \sec\theta = \frac{r}{x} \ (x \neq 0) \qquad \cot\theta = \frac{y}{r} \ (y \neq 0)$$

Quadrantal angles are angles that are coterminal with the coordinate axes.

Let θ be an angle in standard position. The **reference angle** $\bar{\theta}$ associated with θ is the acute angle formed by the terminal side of θ and the x-axis.

To evaluate a trig function for an angle:

1. Find the reference angle $\bar{\theta}$ for the angle θ.
2. Determine the sign of θ by figuring the quadrant in which θ lies.
3. The absolute value of the trig function of θ is the same as $\bar{\theta}$, so you must attach the sign from step 2.

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Even-Odd Identities
Sine, cosecant, tangent, and cotangent are odd functions; cosine and secant and even functions.

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

The **area of a triangle** with sides of lengths a and b and with included angle θ is $A = \frac{1}{2}ab\sin\theta$.

Example 1: Find the reference angle for the following:

-60° 770° $\frac{23\pi}{4}$ $-\frac{56\pi}{5}$

Example 2: Find the exact value of the following trig functions:

tan 135° sin (-660°) $\cot\left(-\frac{23\pi}{4}\right)$ $\sec\frac{62\pi}{3}$

Example 3: Find the quadrant in which θ lies given that cot θ < 0 and csc θ < 0.

Example 4: Find the values of the trig functions of θ given that csc θ = 5 and cos θ < 0.

Example 5: The time in seconds that it takes for a sled to slide down a hillside inclined at an angle θ is $t = \sqrt{\frac{d}{16\sin\theta}}$ where d is the length of the slope in feet. Find the time it takes to slide down a 2500-foot slope inclined at 45°.