

Math 150 Lecture Notes

The Law of Sines

Recall that the following properties are triangle congruences, i.e., there is exactly one possible triangle that can be drawn with the given information.

SSS SAS ASA AAS

An AAA congruence does not produce triangle congruence, although the triangles are similar. A SSA congruence (ASS) does not produce a triangle congruence, since there are sometimes two triangles that meet the given criteria.

To solve a triangle given ASA, AAS, or SSA, use the Law of Sines. To solve a triangle given SAS or SSS, use the Law of Cosines (often followed by the Law of Sines).

Law of Sines

In triangle ABC , $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

When two sides and an angle not between them (SSA) is known, there may or may not be a unique triangle. This situation is often called the **ambiguous case**.

I. If $\angle A$ is acute and $a < b$,

- a. If $b \sin A < a < b$, then 2 Δ 's
- b. If $b \sin A = a$, then 1 Δ .
- c. If $b \sin A > a$, then 0 Δ 's.

II. If $\angle A$ is acute and $a > b$, then 1 Δ .

III. If $\angle A$ is obtuse,

- a. If $a > b$, then 1 Δ .
- b. If $a \leq b$, then 0 Δ .

Example 1: Sketch the triangle and then solve it using Law of Sines.

$$\angle A = 23^\circ \quad \angle B = 100^\circ \quad c = 50$$

Example 2: Sketch the triangle and then solve it using Law of Sines.

$$\angle B = 31^\circ \quad \angle C = 50^\circ \quad b = 45$$

Example 3: Sketch the triangle and then solve it using Law of Sines.

$$a = 50 \quad b = 60 \quad \angle A = 35^\circ$$