

Math 150 Lecture Notes

Trigonometric Identities

Fundamental Trigonometric Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Even-Odd Identities

Sine, cosecant, tangent, and cotangent are odd functions; cosine and secant are even functions.

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\sec(-x) = \sec x$$

$$\cot(-x) = -\cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

To Prove a Trigonometric Identity:

1. Start with one side and try to transform it into the other side using identities. It is usually easier to start with the more complicated side.
2. Use algebra and trig identities to change the side you started with. Rewrite fractions with equivalent fractions using a common denominator. Use factoring and trig identities to simplify expressions.
3. When all else fails, try rewrite all functions in terms of sine and cosine.

Note: You may “work backwards” from the other side, but you **cannot** leave the proof in that form. Use that information only to help you figure out how to think about the proof.

Example 1: Simplify the trig expression: $\frac{\sec x - \cos x}{\tan x}$

Example 2: Simplify the trig expression: $\tan x \cos x \csc x$

Example 3: Prove the identity: $\frac{1 - \sin x}{1 + \sin x} = (\sec x - \tan x)^2$

Example 4: Prove the identity: $\frac{\tan x \sin x}{\tan x + \sin x} = \frac{\tan x - \sin x}{\tan x \sin x}$

Example 5: Prove the identity:

$$\frac{\tan x - \cot x}{\tan^2 x - \cot^2 x} = \sin x \cos x$$