

## Math 150 Lecture Notes

### Inverse Trigonometric Functions

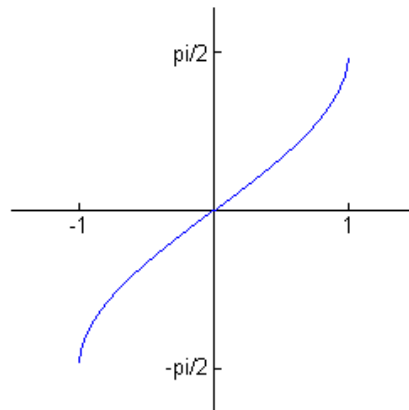
The **inverse sine function** (also called **arcsine** and denoted **arcsin**) is the function  $\sin^{-1}$  with domain  $[-1, 1]$  and range  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  defined by

$$\sin^{-1} x = y \Leftrightarrow \sin y = x$$

So  $\sin^{-1} x$  is the number in the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  whose sine is  $x$ .

The following relations hold true:

$$\begin{aligned} \sin(\sin^{-1} x) &= x && \text{for } -1 \leq x \leq 1 \\ \sin^{-1}(\sin x) &= x && \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \end{aligned}$$



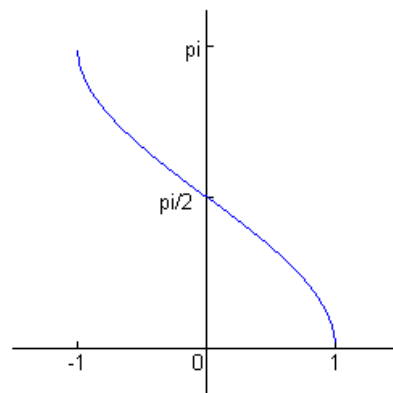
The **inverse cosine function** (also called **arccosine** and denoted **arccos**) is the function  $\cos^{-1}$  with domain  $[-1, 1]$  and range  $[0, \pi]$  defined by

$$\cos^{-1} x = y \Leftrightarrow \cos y = x$$

So  $\cos^{-1} x$  is the number in the interval  $[0, \pi]$  whose cosine is  $x$ .

The following relations hold true:

$$\begin{aligned} \cos(\cos^{-1} x) &= x && \text{for } -1 \leq x \leq 1 \\ \cos^{-1}(\cos x) &= x && \text{for } 0 \leq x \leq \pi \end{aligned}$$



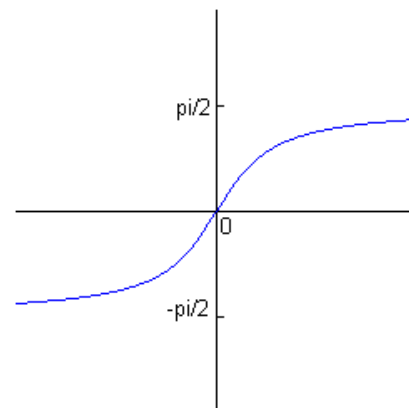
The **inverse tangent function** (also called **arctangent** and denoted **arctan**) is the function  $\tan^{-1}$  with domain  $(-\infty, \infty)$  and range  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  defined by

$$\tan^{-1} x = y \Leftrightarrow \tan y = x$$

So  $\tan^{-1} x$  is the number in the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  whose tangent is  $x$ .

The following relations hold true:

$$\begin{aligned} \tan(\tan^{-1} x) &= x && \text{for } x \in \mathcal{R} \\ \tan^{-1}(\tan x) &= x && \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2} \end{aligned}$$



Example 1: Find the exact value of each expression.

$$\tan^{-1} \frac{\sqrt{3}}{3} \qquad \sin^{-1} 1 \qquad \cos^{-1} \frac{\sqrt{2}}{2} \qquad \tan^{-1} 0$$

Example 2: Use a calculator to find an approximate value of each expression correct to four decimal places.

$$\sin^{-1} (1.4327) \qquad \cos^{-1} (-0.7534) \qquad \tan^{-1} (-2.5732) \qquad \sin^{-1} (-0.8359)$$

Example 3: Find the exact value of each expression.

$$\sin \left( \sin^{-1} \frac{\sqrt{2}}{2} \right) \qquad \sin \left( \cos^{-1} \frac{\sqrt{3}}{2} \right) \qquad \tan^{-1} \left( \sin \left( -\frac{\pi}{6} \right) \right)$$

Example 4: Evaluate the expression by sketching a triangle.

$$\sin (\tan^{-1} 5) \qquad \cot (\sin^{-1} \frac{1}{2}) \qquad \sin (2 \cos^{-1} 0) \qquad \cos \left( \sin^{-1} \frac{4}{5} - \cos^{-1} \frac{4}{5} \right)$$

Example 5: Rewrite the expression as an algebraic expression in  $x$ .

$$\tan (\sin^{-1} x)$$